Quick review T/F questions

True or false. Justify!

1) If P and Q are two Sylow subgroups of a group G, then P and Q intersect trivially. FALSE
2) If P and Q are two Sylow p-subgroups of a group G, then P and Q intersect trivially. FALSE
3) If p and q are distinct primes, P is a Sylow p-subgroup of G and Q is a Sylow q-subgroup of G, then P and Q intersect trivially. TRUE
4) Sylow subgroups are normal. FALSE
5) The direct product of two groups is always abelian. FALSE
6) The semidirect product of two groups is always nonabelian. FALSE
7) There always exists a ring homomorphism between any two rings. FALSE
8) Given any ring R, there exists exactly one ring homomorphism $\mathbb{Z} \longrightarrow R$. TRUE
9) Given any ring R, there exists exactly one ring homomorphism $R \longrightarrow \mathbb{Z}$. FALSE
10) Given any ring R, there exists exactly one ring homomorphism $\mathbb{Z}/n \longrightarrow R$. FALSE
11) Given any ring R, there exists exactly one ring homomorphism $R \longrightarrow \mathbb{Z}/n$. FALSE
12) Every nonzero element in $\mathbb{Z}$ is a unit. FALSE
13) Every ring contains at least two ideals. TRUE
14) Every domain is a field. FALSE
15) Every field is a domain. TRUE
16) Any ring that has only two ideals is a field. FALSE
17) The ring $\mathbb{Z}/n[x]$ is a domain. FALSE
18) If R and S are domains, then $R \times S$ is a domain. FALSE
19) Any subring of a domain is a domain. TRUE
20) Any subring of a field is a field. FALSE
21) The kernel of any ring homomorphism is an ideal of the domain. TRUE
22) The kernel of any ring homomorphism is a subring of the domain. FALSE
23) The image of any ring homomorphism is an ideal of the codomain. FALSE
24) The image of any ring homomorphism is a subring of the codomain. TRUE
25) Every proper ideal is the kernel of some ring homomorphism. TRUE
26) If R is a commutative ring and $(g) = R$, then g is a unit. TRUE
27) If R is a domain, then $R[x]$ is a domain. TRUE
28) If F is a field, then $F[x]$ is a field. FALSE
29) If $p \in \mathbb{Z}/2[x]$ has degree 3, then $\mathbb{Z}/2[x]/(p)$ has 4 elements. FALSE
30) If $p \in F[x]$ for some field F is irreducible, then $\gcd(p, f)$ is 1 or p. TRUE
31) In $R[x]$, the product of two nonzero polynomials can be zero. TRUE
32) If $uf + vg = 4$ in $\mathbb{Q}[x]$, then $f + (g)$ is a unit in $\mathbb{Q}[x]/(g)$. TRUE
33) The element $x^2 + 4 + (x^4 - x^2) \in \mathbb{Z}/5[x]/(x^4 - x^2)$ is a unit. FALSE
34) The element $x^3 + 2 + (x^4 - x^2) \in \mathbb{Z}/5[x]/(x^4 - x^2)$ is a unit. TRUE
35) The quotient ring $\mathbb{R}[x]/(x^3 - x - 6)$ is a field. FALSE
36) An element of a commutative ring R can be both a unit and a zerodivisor. FALSE
37) $\mathbb{Z}/n$ is a domain if and only if it is a field. TRUE
38) Every nonzero element in $\mathbb{Z}/21$ is a unit. FALSE
39) In $\mathbb{Z}/77$, $(a) = (b)$ if and only if $a = b$. FALSE
40) Every ideal in $\mathbb{Z}/123$ is principal. TRUE
41) In any ring R, $ab = 0$ implies $a = 0$ or $b = 0$. FALSE
42) In any ring R, we can cancel addition: $a + b = a + c \implies b = c$. TRUE
43) In any ring R, we can cancel multiplication: $ab = ac \implies b = c$. FALSE
44) If I and J are ideals in a ring R, $I \cup J$ is an ideal in R. FALSE