Problem Set 8
Due Wednesday, November 13

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand. You cannot use any resources besides me, your classmates, and our course notes.

I will post the .tex code for these problems for you to use if you wish to type your homework. If you prefer not to type, please *write neatly*. As a matter of good proof writing style, please use complete sentences and correct grammar. You may use any result stated or proven in class or in a homework problem, provided you reference it appropriately by either stating the result or stating its name (e.g. the definition of ring or Lagrange's Theorem). Please do not refer to theorems by their number in the course notes, as that can change.

Problem 1. Show that there are no simple groups of order $56 = 8 \cdot 7$.

Problem 2. Show that there are no simple groups of order $2^5 \cdot 7^3$.

Problem 3. Let G be a finite group of order pqr with $0 < p < q < r$ prime numbers. Show that G is not simple.

Problem 4. Prove that S_4 has precisely three distinct subgroups of order 8, all of which are isomorphic to D_4.

Problem 5. Let C_n denote the cyclic group of order $n \geqslant 2$, and consider the group

$$(\mathbb{Z}/n)^\times = \{[j]_n \mid \gcd(j, n) = 1\}$$

with the binary operation given by the usual multiplication. Prove that

$$\mathrm{Aut}(C_n) \cong (\mathbb{Z}/n)^\times.$$