Quick review T/F questions

True or false. Justify!

1) $D_3 \cong \mathbb{Z}_2 \times \mathbb{Z}_3$.
2) $D_3 \cong \mathcal{S}_3$.
3) $D_4 \cong \mathcal{S}_4$.
4) $\mathbb{Z}_2 \times \mathbb{Z}_2 \cong \mathbb{Z}_4$.
5) $\mathbb{Z}_3 \times \mathbb{Z}_2 \cong \mathbb{Z}_6$.
6) Let R be the subgroup of all rotations in D_4. Then $D_4/R \cong \mathbb{Z}_3^\times$.
7) If G and H are two groups of the same order, then $G \cong H$.
8) Every finite group G is isomorphic to a subgroup of $\mathcal{S}_n$ for some n.
9) There are two nonisomorphic cyclic groups of order 20.
10) An element g of a group G can satisfy $g^{30} = e$ and have order 6.
11) An element g of a group G can satisfy $g^{30} = e$ and have order 7.
12) If a group G contains an element of infinite order, then G is infinite.
13) If a group G contains a nontrivial element of finite order, then G is finite.
14) If every element in a group G has finite order, then G is finite.
15) If G is a group of order n and $k|n$, there is an element of G of order k.
16) If G is a group of order n and $k|n$, there is a subgroup of G of order k.
17) Every group of order 12 contains an element of order 4.
18) In any group G, the product of elements of finite order always has finite order.
19) Every 4-cycle in $\mathcal{S}_{103}$ is odd.
20) Every element in $\mathcal{S}_{123}$ is a product of elements of order 2.
21) Every element in $\mathcal{S}_{123}$ is a product of elements of order 3.
22) $\mathcal{S}_3$ is a cyclic group.
23) There always exists a group homomorphism between any two groups.
24) There exists a surjective group homomorphism $\mathbb{Z}_7 \longrightarrow \mathbb{Z}_5$.
25) There exists an injective group homomorphism $\mathcal{S}_7 \longrightarrow \mathcal{S}_8$.
26) There exists an injective group homomorphism $\mathbb{Z}_7 \longrightarrow \mathbb{Z}_8$.
27) The image if a group homomorphism $G \longrightarrow H$ with G abelian is always an abelian subgroup of H.
28) If there exists a nontrivial group homomorphism $G \longrightarrow H$ with G abelian, then H is abelian.
29) If H is an abelian subgroup of the (possibly nonabelian) group G, then H is normal.
30) If H is a subgroup of an abelian group G, then G/H is abelian.
31) If every proper subgroup of a group G is cyclic, then G is cyclic.
32) There exists a subgroup of S_5 that is isomorphic to $\mathbb{Z}_3 \times \mathbb{Z}_3$.
33) Every nontrivial group has at least two subgroups.
34) Every subgroup of an abelian group is abelian.
35) The center of an abelian group G is the set of all elements of G.
36) The center of a group G is an abelian group.
37) The center of a group is always a normal subgroup.
38) A subgroup of order 2 is always normal.
39) Every subgroup of index 2 is normal.
40) Every subgroup of index 3 is normal.
41) The image of a group homomorphism is always a normal subgroup.
42) The kernel of a group homomorphism is always a normal subgroup.
43) Every nontrivial group has at least two normal subgroups.
44) Every normal subgroup is the kernel of some group homomorphism.

45) The intersection of two normal subgroups is a normal subgroup.
46) The quotient group G/K is a subset of G.
47) The elements gK and hK are equal in G/K if and only if $g = h$.
48) The quotient group $\mathbb{Q}/\mathbb{Z}$ is a finite group.
49) Every quotient of a nonabelian group is nonabelian.
50) There exists a surjective group homomorphism $\mathcal{S}_5 \longrightarrow \mathcal{S}_4$.