Quick review T/F questions

True or false. Justify!

1) $D_3 \cong \mathbb{Z}_2 \times \mathbb{Z}_3$. FALSE
2) $D_3 \cong \mathcal{S}_3$. TRUE
3) $D_4 \cong \mathcal{S}_4$. FALSE
4) $\mathbb{Z}_2 \times \mathbb{Z}_2 \cong \mathbb{Z}_4$. FALSE
5) $\mathbb{Z}_3 \times \mathbb{Z}_2 \cong \mathbb{Z}_6$. TRUE
6) Let R be the subgroup of all rotations in D_4. Then $D_4/R \cong \mathbb{Z}_3^\times$. TRUE
7) If G and H are two groups of the same order, then $G \cong H$. FALSE
8) Every finite group G is isomorphic to a subgroup of $\mathcal{S}_n$ for some n. TRUE
9) There are two nonisomorphic cyclic groups of order 20. FALSE
10) An element g of a group G can satisfy $g^{30} = e$ and have order 6. TRUE
11) An element g of a group G can satisfy $g^{30} = e$ and have order 7. FALSE
12) If a group G contains an element of infinite order, then G is infinite. TRUE
13) If a group G contains a nontrivial element of finite order, then G is finite. FALSE
14) If every element in a group G has finite order, then G is finite. FALSE
15) If G is a group of order n and $k|n$, there is an element of G of order k. FALSE
16) If G is a group of order n and $k|n$, there is a subgroup of G of order k. FALSE
17) Every group of order 12 contains an element of order 4. FALSE
18) In any group G, the product of elements of finite order always has finite order. FALSE
19) Every 4-cycle in $\mathcal{S}_{103}$ is odd. TRUE
20) Every element in $\mathcal{S}_{123}$ is a product of elements of order 2. TRUE
21) Every element in $\mathcal{S}_{123}$ is a product of elements of order 3. FALSE
22) $\mathcal{S}_3$ is a cyclic group. FALSE
23) There always exists a group homomorphism between any two groups. TRUE
24) There exists a surjective group homomorphism $\mathbb{Z}_7 \longrightarrow \mathbb{Z}_5$. FALSE
25) There exists an injective group homomorphism $\mathcal{S}_7 \longrightarrow \mathcal{S}_8$. TRUE
26) There exists an injective group homomorphism $\mathbb{Z}_7 \longrightarrow \mathbb{Z}_8$. FALSE
27) The image if a group homomorphism $G \longrightarrow H$ with G abelian is always an abelian subgroup of H. TRUE
28) If there exists a nontrivial group homomorphism $G \longrightarrow H$ with G abelian, then H is abelian. FALSE
29) If H is an abelian subgroup of the (possibly nonabelian) group G, then H is normal. FALSE
30) If H is a subgroup of an abelian group G, then G/H is abelian. TRUE
31) If every proper subgroup of a group G is cyclic, then G is cyclic. FALSE
32) There exists a subgroup of S_5 that is isomorphic to $\mathbb{Z}_3 \times \mathbb{Z}_3$. FALSE
33) Every nontrivial group has at least two subgroups. TRUE
34) Every subgroup of an abelian group is abelian. TRUE
35) The center of an abelian group G is the set of all elements of G. TRUE
36) The center of a group G is an abelian group. TRUE
37) The center of a group is always a normal subgroup. TRUE
38) A subgroup of order 2 is always normal. FALSE
39) Every subgroup of index 2 is normal. TRUE
40) Every subgroup of index 3 is normal. FALSE
41) The image of a group homomorphism is always a normal subgroup. FALSE
42) The kernel of a group homomorphism is always a normal subgroup. TRUE
43) Every nontrivial group has at least two normal subgroups. TRUE

44) Every normal subgroup is the kernel of some group homomorphism. TRUE
45) The intersection of two normal subgroups is a normal subgroup. TRUE
46) The quotient group G/K is a subset of G. FALSE
47) The elements gK and hK are equal in G/K if and only if $g = h$. FALSE
48) The quotient group $\mathbb{Q}/\mathbb{Z}$ is a finite group. FALSE
49) Every quotient of a nonabelian group is nonabelian. FALSE
50) There exists a surjective group homomorphism $\mathcal{S}_5 \longrightarrow \mathcal{S}_4$. FALSE