depth(R) ≤ dim(R)   More generally, depth(M) ≤ dim(R)

(R, m) Noetherian local ring

R is Cohen–Macaulay if depth(R) = dim(R)

M is Cohen–Macaulay if depth(M) = dim(M)

Examples

- Every regular ring is Cohen–Macaulay
- Every 1-dimensional domain is Cohen–Macaulay
- $k[x]/(x^2)$ is Cohen–Macaulay but not regular
- $k[x,y,z]/(xy,yz)$ is not Cohen–Macaulay
- Rings with nice singularities are Cohen–Macaulay

eg

rings of invariants of a finite group G acting on $k[x_1, \ldots, x_d]$, char $k \nmid |G|$

depth R ≤ dim(R) ≤ embdim(R)

(↓ = Cohen–Macaulay; ↓ = regular)

- M Cohen–Macaulay

$\Rightarrow$ depth(M) = dim(R/P) for all $P \in \mathrm{Ass}(M)$

$\Rightarrow$ M has no embedded primes

- Cohen–Macaulayness localizes

- R Cohen–Macaulay, P prime

dim(R) − height(P) = dim(R/P)

Def R is Cohen-Macaulay if R_I is Cohen-Macaulay for all primes I.

- theorem R Noetherian ring

R is Cohen-Macaulay $\iff$ every ideal I contains a regular sequence of length $ht(I)$

∴ Over a Cohen-Macaulay ring

I is generated by a regular sequence $\iff$ I generated by $ht(I)$ elements

Regular $\implies$ Cohen-Macaulay

eg all $k[x_1, \dots, x_d]$ are Cohen-Macaulay

R Cohen-Macaulay
$x_1, \dots, x_n$ regular sequence $\implies$ $R/(x_1, \dots, x_n)$ is Cohen-Macaulay

eg regular ring / regular sequence is Cohen-Macaulay

complete intersections, eg hypersurfaces