Regular rings (R, m, k) regular local ring
if $\dim(R) = d$ and $\mu(m) = d$

Idea Geometrically, think about varieties of dimension d that embed on $\mathbb{A}^d_k$
(so very nice and smooth)

theorem (Auslander – Buchsbaum, Serre)
(R, m, k) Noetherian local ring of dim d. TFAE:

① $\operatorname{pdim}_R(k) < \infty$
② $\operatorname{pdim}_R(M) < \infty$ for all fg R-module M
③ m is generated by a regular sequence
④ m is generated by d elements
$\quad\quad\quad\quad$ ||
(R, m, k) regular local ring

A ring R is regular if R_I is regular for every prime I

- Every regular local ring is a domain
- R RLR $\Rightarrow$ $\operatorname{pdim}_R(k) = \dim(R)$
- every PID is a regular ring
- R regular local ring $\Rightarrow R_I$ regular

Example $R = k[x_1, \dots, x_n]$ is a regular ring

R regular $\Longleftrightarrow$ Every fg R-module has finite pdim ($\leq \dim R$)

Cohen structure theorem R Noetherian local ring
then $\hat{R} \cong Q/I$ for some regular local ring Q