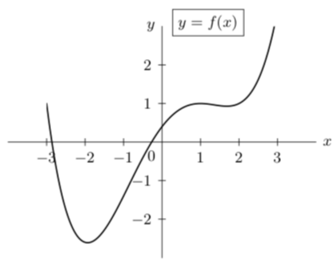
Worksheet 10

Warm-up questions

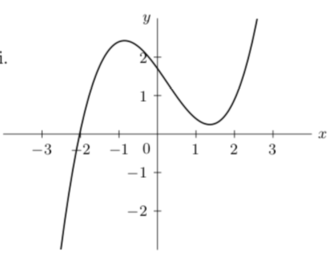
If $f'(x) < 0$ on an interval, then f is ________________ on that same interval.
If $f'(x) > 0$ on an interval, then f is ________________ on that same interval.
If f is constant on an interval, then $f'(x) =$________________ for all values of x in that interval.
If f is linear on an interval, then $f'(x) =$________________ for all values of x in that interval.

Problem 1 (Fall 2015 Exam 1)**.** Below is the graph of a function f.

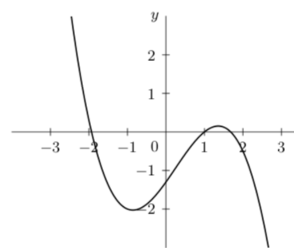


There are six graphs shown below. Circle the one graph that could be the graph of the derivative $f'(x)$.

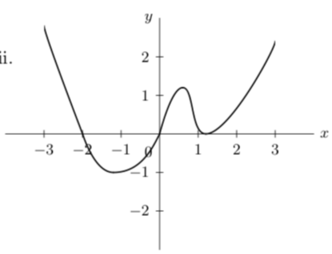
i.



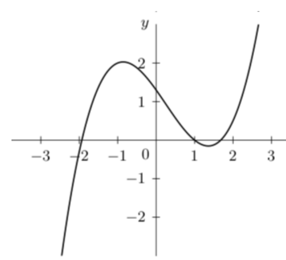
iv.



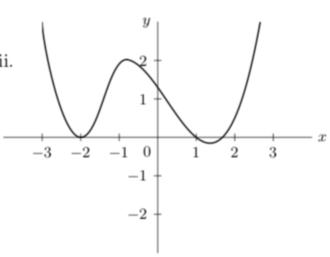
ii.



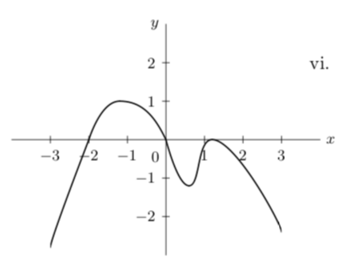
v.



iii.

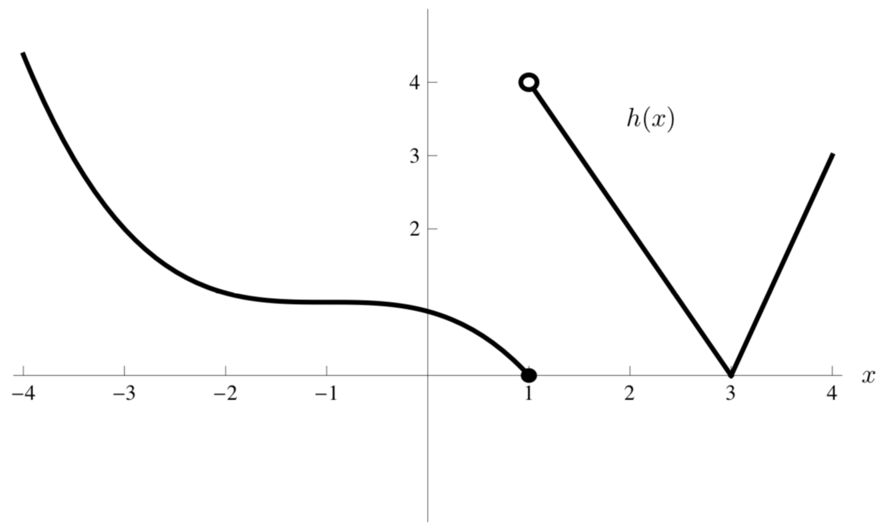


vi.



Problem 2 (Winter 2016 Exam 1)**.** The graph of the function $h(x) = (x+3)e^{2x-2}$ is given by the formula $h'(x) = (2x+7)e^{2x-2}$. Find an equation for the tangent line to the graph of $y = h(x)$ at $x = 1$.

Problem 3 (Winter 2013 Exam 1)**.** Below is the graph of a function h.



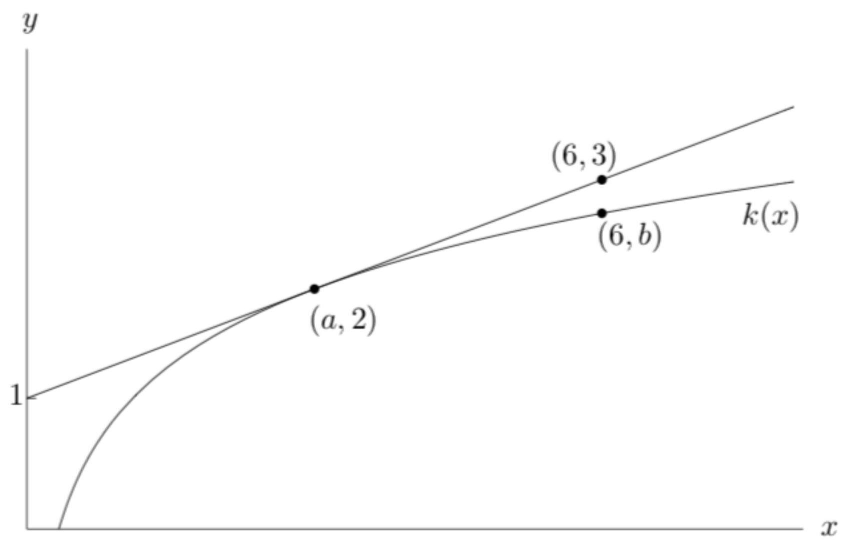
Carefully draw a graph of $h'(x)$. Be sure to label important points or features on your graph.

Problem 4 (Winter 2017 Exam 1)**.** Sketch the graph of a single function $y = f(x)$ satisfying all of the following conditions:

- The domain of f(x) is the interval $-8 < x \leqslant 6$.
- $f(x)$ is continuous for all x in the interval $-8 < x < -2$.
- $f'(-7) = 0$.
- $f(x)$ is decreasing and concave up for all x in the interval $-6 < x < -4$.
- The average rate of change of $f(x)$ is equal to 0.5 between $x = -5$ and $x = -2$
- $f(0) = 2$ and $f'(0) = -1$.
- $\lim_{x \to 2^-} f(x) = f(2)$ and $\lim_{x \to 2^+} f(x) < \lim_{x \to 2^-} f(x)$.
- $f(x)$ has constant rate of change in the interval $3 \leqslant x \leqslant 6$.

Make sure that your graph is large and unambiguous.

Problem 5 (Winter 2013 Exam 1)**.** The figure on the next page shows the graph a function $k(x)$ and its tangent line at a point $(a, 2)$. The average rate of change of $k(x)$ between $x = a$ and $x = 6$ is $\frac{1}{6}$. Find exact numerical values for the following. If it is not possible to find a value, write **NP**. You do not need to show your work.



(a) $a =$ (c) $k'(2) =$ (e) $k'(6) =$

(b) $b =$ (d) $k'(a) =$

Problem 6 (Fall 2017 Exam 1)**.** Sketch the graph of a function $y = R(x)$ verifying all of the following conditions:

- The function $R(x)$ is defined on $-8 \leqslant x \leqslant 9$.
- $R'(x) = 2$ for $-8 < x < -5$.
- $R(x)$ is concave down and decreasing on $-5 < x < 2$.
- $R(-2) = 1$.
- $R(x) = R(-x)$ for $-2 \leqslant x \leqslant 2$.
- The vertical intercept of $R(x)$ is $y = 3$.
- $\lim_{x\to 5^-} R(x) = -2$ but $\lim_{x\to 5^+} R(x)$ does not exist.
- $R(x)$ is not continuous at $x = 7$ but $\lim_{x\to 7} R(x)$ exists.

Make sure that your graph is large and unambiguous.

Problem 7 (Winter 2013 Exam 1)**.** In each of the following problems, give a formula for a function whose domain is all real numbers, with all of the indicated properties. If there is no such function, then write NO SUCH FUNCTION EXISTS. You do not need to show your work.

(a) A sinusoidal function $P(t)$ with the following three properties:

- The period of the graph of $P(t)$ is 7.
- The graph of $P(t)$ attains a maximum value at the point $(1, 20)$.
- The graph of $P(t)$ attains a minimum value at the point $(-2.5, 6)$.

(b) A function $h(x)$ with the following two properties:

- $h(x)$ is concave down for all x.
- $h(x) > 0$ for all x.

(c) A function $j(x)$ with the following two properties:

- $j(x)$ is decreasing for all x.
- $j(x)$ is concave up for all x.

(d) A rational function $l(x)$ with the following two properties:

- $l(0) = 2$.
- The line $y = 2$ is a horizontal asymptote to the graph of $l(x)$.

Problem 8 (Fall 2009 Exam 1)**.** If $f(x) = \frac{g(x)}{h(x)}$ and $h(3) = 0$, which of the following statements MUST be true?

(a) The graph of f has a vertical asymptote at $x = 3$.

(b) 3 is not in the domain of f.

(c) f is not continuous on $[-2, 2]$.

(d) $\lim_{x \to 3} f(x)$ does not exist.