Worksheet 11

Practical interpretation of the derivative

Example 1: The temperature, T, in degrees Fahrenheit, of a cold yam placed in a hot oven is given by $T = f(t)$, where t is the time in minutes since the yam was put in the oven. Give a practical interpretation of the equation $f'(20) = 2$ in the context of this problem that can be understood by someone who knows no calculus. Write a complete sentence, and be sure to include units in your answer.

Possible answers:

- During its 21st minute in the oven, the temperature of the yam increases by approximately 2°F.
- After the yam has been in the oven for 20 minutes, the yam's temperature will increase by about 1/3°F if it stays in the oven an additional 10 seconds.

Example 2: The number of Wooters (registered members of Woot.com) is currently over 500,000. Since there is not a mechanism for "un-registering“, and the membership has grown very quickly, assume that the number of Wooters, W in thousands, is an invertible function of time, t, in hours, $W = g(t)$. In this context, give a practical interpretation of $(g^{-1})'(200) = 0.05$.

Possible answers:

- It took approximately 3 minutes for the number of Wooters to increase from 200,000 to 201,000.
- When there were 200,000 registered members of Woot.com, it took approximately 3 minutes for the next 1000 Wooters to register.
- When there were 200,000 registered members of Woot.com, it took about 90 seconds for the next 500 Wooters to register.

Key features of a good answer:

- A complete, comprehensible English sentence that could be understood by someone who does not know any calculus. Avoid words like *rate*.
- Stating the given input value and indicating a small change in the independent variable from that input value.
- Indicating that the change in the output is approximate, using words like *roughly*, *approximately* or *about*.
- Indicating the appropriate direction of change of output (increase versus decrease).
- Using appropriate numbers and units.

Problem 1 (Fall 2015 Exam 1)**.** Algernon Brayik is making scones. He knows that the height of a scone is a function of how much baking soda it contains. Let h(B) be the height in millimeters of a scone that contains B grams of baking soda. Assume that the function h is increasing and invertible, and that h and h1 are both differentiable.

(a) Algie looks in his baking soda container and finds that there are exactly 46 grams of baking soda remaining. Suppose he uses all of this baking soda to make 8 scones, and that the baking soda is equally distributed among all 8 of the scones. Write a mathematical expression involving h or h^{-1} for the height (in millimeters) of each resulting scone.

(b) Below is the first part of a sentence that will give a practical interpretation of the equation $h'(6) = 15$ in the context of this problem. Complete the sentence so that the practical interpretation can be understood by someone who knows no calculus. Be sure to include units in your answer.

If Algie decreases the amount of baking soda per scone from 6 grams to 5.8 grams, then...

Algie makes a batch of scones, with each scone containing k grams of baking soda (for some constant k). When the scones come out of the oven, he decides they are each 10 millimeters shorter than he would like. Write a mathematical expression involving k, h, and h^{-1} for the number of grams of baking soda per scone he should use to get scones of the desired height.

(c) Algie does some calculations and determines that $\dfrac{60}{h^{-1}(30)} = 40$. Based on this information, which of the following statements must be true? Circle all of the statements that must be true or circle none of these.

A. If Algie makes 40 scones, each with 30 grams of baking soda, then the scones will rise to a height of 60 millimeters.

B. If Algie wants to make 40 scones, then he must use 60 grams of baking soda.

C. If Algie wants to make scones of height 30 millimeters and he has 60 grams of baking soda, then the maximum number of scones he can make is 40.

D. A scone containing 1.5 grams of baking soda rises to a height of 30 millimeters.

E. A scone containing 30 grams of baking soda rises to a height of 1.5 millimeters.

F. None of these

Problem 2 (Winter 2010 Exam 1)**.** Suppose that $W(h)$ is an invertible function which tells us how many gallons of water an oak tree of height h feet uses on a hot summer day.

(a) Give a practical interpretations for each of the following quantities or statements that can be understood by someone who knows no calculus. Be sure to include units in your answer.

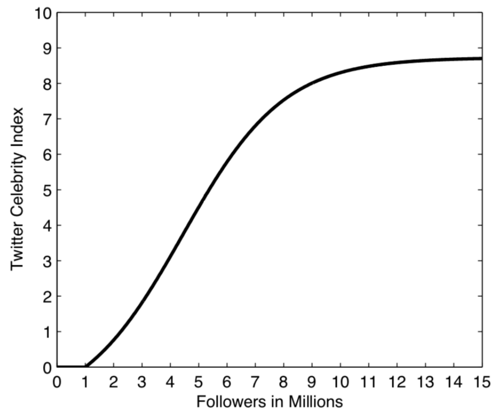
(i) $W(50)$. (ii) $W^{-1}(40)$. (iii) $W'(5)$.

(b) Suppose that an average oak tree is A feet tall and uses G gallons of water on a hot summer day. Answer each of the questions below in terms of the function W. You may also use the constants A and/or G in your answers.

(i) A farmer has a grove with 25 oak trees, and each one is 10 feet taller than an average oak tree. How much water will be used by his trees during a hot summer day?

(ii) The farmer also has some oak trees which each use 5 fewer gallons of water on a hot summer day than an average oak tree does. How tall is one of these trees?

Problem 3 (Fall 2011 Exam 1)**.** The Twitter Celebrity Index (TCI) measures the celebrity of Twitter users; the function T(x) takes the number of followers (in millions) of a given user and returns a TCI value from 0 to 10. Below is a graph of this function.



Use the graph above to help you answer the following questions. Give a practical interpretation of each equation in the context of this problem that can be understood by someone who knows no calculus.

(a) $T(13.72) = 8.67$.

(b) $T^{-1}(4.25) = 4.88$.

(c) $T'(10) = 0.2278$.

(d) $(T^{-1})'(7.238) = 0.71$.

Problem 4 (Fall 2012 Exam 1)**.** Toby listens to music as he walks to class in the morning and notices an interesting phenomenon: the tempo of the music affects his walking speed and thus the time it takes him to get to class. Let $C(b)$ be the number of minutes it takes Toby to get to class when he is listening to music with a tempo of b beats per minute (bpm). You may assume Toby's house is 1.2 miles from his first class. For (a), (b) and (c) below, write a single mathematical equation using C, C^{-1}, and/or their derivatives that describes the given situation.

(a) The tempo of the music Toby is listening to when it takes him 32 minutes to get to class is 89 bpm.

(b) If Toby gets to class in 30 minutes, and he wants to take 31 minutes to get there instead, he should decrease the tempo of his music by approximately 4 bpm.

(c) Toby's average velocity when he listens to music with a tempo of 115 bpm is 0.047 miles per minute.

(d) Give a practical interpretation of $C'(81) = -0.5$ that can be understood by someone who knows no calculus.

Problem 5. Oren, a Math 115 student, realizes that the more caffeine he consumes, the faster he completes his online homework assignments. Before starting tonights assignment, he buys a cup of coffee containing a total of 100 milligrams of caffeine. Let $T(c)$ be the number of minutes it will take Oren to complete tonights assignment if he consumes c milligrams of caffeine. Suppose that T is continuous and differentiable.

(a) Circle the one sentence below that is best supported by the following statement: the more caffeine he consumes, the faster he completes his online homework assignments.

  (i) $T'(c) \geqslant 0$ for every value c in the domain of T.
  (ii) $T'(c) \leqslant 0$ for every value c in the domain of T.
  (iii) $T'(C) = 0$ for every value c in the domain of T.

(b) Explain, in the context of this problem, why it is reasonable to assume that $T(c)$ is invertible.

(c) Interpret the equation $T^{-1}(100) = 45$ in the context of this problem in a way that can be understood by someone who knows no calculus. Use a complete sentence and include units.

(d) Suppose that p and k are constants. In the equation $T'(p) = k$, what are the units on p and k?

(e) Which of the statements below is best supported by the equation $(T^{-1})'(20) = -10$? Circle the one best answer.

  (i) If Oren has consumed 20 milligrams of caffeine, then consuming an additional milligram of caffeine will save him about 10 minutes on tonight's assignment.
  (ii) The amount of caffeine that will result in Oren finishing his homework in 21 minutes is approximately 10 milligrams greater than the amount of caffeine that Oren will need in order to finish his homework in 20 minutes.
  (iii) The rate at which Oren is consuming caffeine 20 minutes into his homework assignment is decreasing by 10 milligrams per minute.
  (iv) In order to complete tonight's assignment in 19 rather than 20 minutes, Oren needs to consume about 10 milligrams of additional caffeine.
  (v) If Oren consumes 20 milligrams of caffeine, then he will finish tonight's assignment approximately 10 minutes faster than if he consumes no caffeine.

Problem 6 (Winter 2012 Exam 1)**.** A certain company's revenue R (in thousands of dollars) is given as a function of the amount of money a (in thousands of dollars) they spend on advertising by $R = f(a)$. Suppose that f is invertible.

(a) Which of the following is a valid interpretation of the equation $(f^{-1})'(75) = 0.5$? Circle one option.

  (i) If the company spends \$75,000 more on advertising, their revenue will increase by about \$500.
  (ii) If the company increases their advertising expenditure from \$75,000 to \$76,000, their revenue will increase by about \$500.
  (iii) If the company wants a revenue of \$75,000, they should spend about \$500 on advertising.
  (iv) If the company wants to increase their revenue from \$75,000 to \$76,000, they should spend about \$500 more on advertising.

(b) The company plans to spend about \$100,000 on advertising. If $f'(100) = 0.5$, should the company spend more or less than \$100,000 on advertising? Justify your answer.

(c) The company's financial advisor claims that he has a formula for the dependence of revenue on advertising expenditure, and it is

$$f(a) = a\ln(a+1).$$

Using this formula, write the limit definition of $f'(100)$. You do not need to simplify or evaluate.