Worksheet 12

Warm-up questions

If $f''(x) < 0$ on an interval, then f is _____ on that same interval.
If $f''(x) > 0$ on an interval, then f is _____ on that same interval.
If the graph of f is _____ on an interval where f'' exists, then $f''(x) \geqslant 0$ on that interval.
If the graph of f is _____ on an interval where f'' exists, then $f''(x) \leqslant 0$ on that interval.
The instantaneous acceleration at a point is _____.

Problem 1 (Fall 2015, Exam 1). Below is the graph of $f'(x)$, the **<u>derivative</u>** of the function $f(x)$.
Note that $f'(x)$ is zero for $x \leqslant -2$, linear for $-2 < x < -1$, and constant for $-1 < x < 0$.
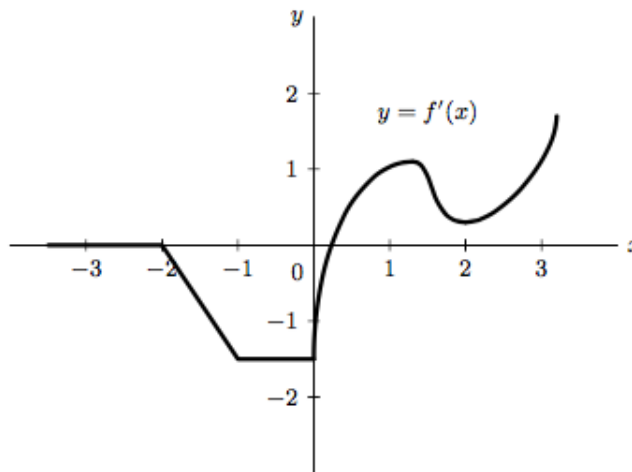
For each of the following, circle <u>all</u> of the listed intervals for which the given statement is true over the entire interval. If there are no such intervals, circle NONE. You do not need to explain your reasoning.

(a) $f'(x)$ is increasing.

$$-2 < x < -1 \qquad 0 < x < 1 \qquad 1 < x < 2 \qquad 2 < x < 3 \qquad \text{NONE}$$

(b) $f'(x)$ is concave up.

$$0 < x < 1 \qquad 1 < x < 2 \qquad 2 < x < 3 \qquad \text{NONE}$$

(c) $f(x)$ is increasing.

$$-2 < x < -1 \qquad -1 < x < 0 \qquad 0 < x < 1 \qquad 1 < x < 2 \qquad 2 < x < 3 \qquad \text{NONE}$$

(d) $f(x)$ is linear but not constant.

$$-3 < x < -2 \qquad -2 < x < -1 \qquad -1 < x < 0 \qquad 0 < x < 1 \qquad 1 < x < 2 \qquad 2 < x < 3 \qquad \text{NONE}$$

(e) $f(x)$ is constant.

$-3 < x < -2$ $-2 < x < -1$ $-1 < x < 0$ $0 < x < 1$ $1 < x < 2$ $2 < x < 3$ NONE

Problem 2 (Winter 2017 Exam 2)**.** The graph of a portion of the derivative of $b(x)$ is shown below. Assume that $b(x)$ is defined and continuous on $[-5, 6]$.
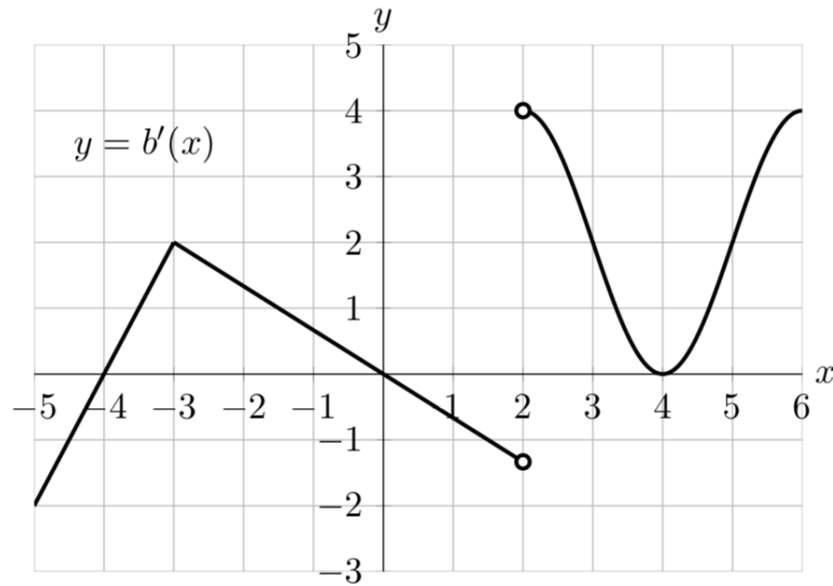
For what values of x is $b(x)$ concave up? Write your answer using inequalities or interval notation.

Problem 3 (Winter 2015 Exam 3)**.** Shown on the axes below are the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Determine which graph is which.
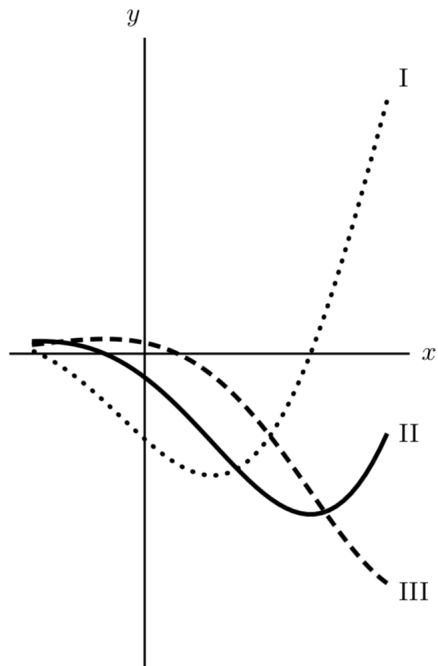
Problem 4 (Winter 2017 Exam 2). Some information about a function $f(x)$ is given in the table below.

x	-2	-1	0	1	2	3	4
$f'(x)$	-2	0	-2	0	1	0	-1
$f''(x)$	1	0	0	2	0	0	-2

Assume that $f''(x)$ is continuous on $[-2, 4]$ and that the values of $f'(x)$ and $f''(x)$ are strictly positive or strictly negative between consecutive table entries. Circle all of the intervals on which $f''(x)$ must be negative, if any exist.

$02 < x < -1$ $-1 < x < 0$ $0 < x < 1$ $1 < x < 2$ $2 < x < 3$ $3 < x < 4$ None of these

Problem 5 (Winter 2016 Exam 1). The graph of a function f is shown below.
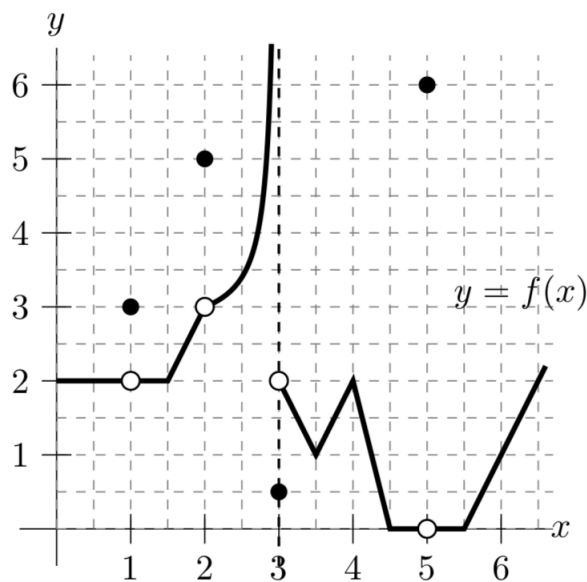
Note: You may assume that pieces of the function that appear linear are indeed linear. Use the graph above to evaluate each of the expressions below.

(a) $f(1)$

(b) $\lim\limits_{x \to 5} f(x)$

(c) $\lim\limits_{q \to 3} f(q)$

(d) $\lim\limits_{z \to 2} f(2)$

(e) $\lim\limits_{r \to 6^-} f(r)$

(f) $\lim\limits_{h \to 20} \dfrac{f(4.25 + h) - f(4.25)}{h}$

(g) $\lim\limits_{p \to 0.5} \dfrac{f(p)}{p}$

(h) $\lim\limits_{t \to 3} f(t)f(t+2)$

(i) $\lim\limits_{x \to 3^+} f(f(x))$

(j) $\lim\limits_{s \to 1} f(f(s))$