Worksheet 14

Warm-up question

$(x^n)' =$

Given functions f and g and $h(x) = f(x) + g(x)$, $h'(x) =$

Problem 0. Compute the derivatives of the functions in Exercises 6–24 in section 3.1.

Problem 1. The graph of $f(x) = x^3 - 9x^2 - 16x + 1$ has two points where the tangent line to the graph has a slope of 5. Find the coordinates of those points.

Problem 2. On what intervals is the graph of $g(x) = x^4 - 4x^3$ both decreasing and concave up?

Problem 3. For what values of x is the function $f(x) = x^5 - 5x$ both increasing and concave up?

Problem 4. The n^{th} derivative of f, $f^{(n)}(x)$, is the result of differentiating $f(x)$ n times. Consider the function $f(x) = x^7 + 5x^5 - 4x^3 + 6x - 7$.

(a) Find the *8th* derivative of $f(x)$. Think ahead!

(b) Find the 7th derivative of $f(x)$.

Problem 5. Suppose p is a cubic polynomial function, meaning that $p(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ for some constants a_0, a_1, a_2, a_3, with $a_0 \neq 0$.

(a) Write expressions for $p(0)$, $p'(0)$, $p''(0)$ and $p'''(0)$ depending on a_0, a_1, a_2, and a_3.

(b) Find the formula for a cubic polynomial function q that satisfies

$$q(0) = 2, \quad q'(0) = -1, \quad q''(0) = 5, \quad q'''(0) = 4.$$

Problem 6. Assume that f'' and g'' exist and that f and g are concave up for all x. Are the following statements true or false? If a statement is true, explain how you know. If a statement is false, give a counterexample.

(a) $f(x) + g(x)$ is concave up for all x.

(b) $f(x) - g(x)$ cannot be concave up for all x.

Problem 7. Let $f(x) = x^4 - 3x^2 + 1$.

(a) Show that $f(x)$ is an even function.

(b) Show that $f'(x)$ is an odd function.

(c) Are all polynomials of even degree even functions?