Worksheet 15

Warm-up question

$(e^x)' =$ $(a^x)' =$ $(\sin(x))' =$ $(\cos(x))' =$

Problem 1. Find a quadratic polynomial $p(x) = ax^2+bx+c$ which best fits the function $f(x) = e^x$ at $x = 0$, in the sense that $p(0) = f(0)$, $p'(0) = f'(0)$ and $p''(0) = f''(0)$.

Problem 2. Find the 50^{th} derivative of $y = \cos(x)$.

Problem 3. Find the derivatives of the functions. Assume that a, b, c, and k are constants.

(a) $f(x) = 2e^x + x^2$

(b) $f(x) = 12e^x + 11^x$

(c) $y = 4 \cdot 10^x - x^3$

(d) $y = 2^x + \dfrac{2}{x^3}$

(e) $y = 5 \cdot 5^t + 6 \cdot 6^t$

(f) $y = \pi^2 + \pi^x$

(g) $f(x) = e^k + k^x$

(h) $f(\theta) = e^{k\theta} - 1$

(i) $f(x) = 2x - \dfrac{1}{\sqrt[3]{x}} + 3^x - e$

(j) $y = 5t^2 + 4e^t$

(k) $y = 5x^2 + 2^x + 3$

(l) $z = (\ln 4)e^x$

(m) $y = (\ln 4)4^x$

(n) $h(z) = (\ln 2)^z$

(o) $f(x) = e^\pi + \pi^x$

(p) $f(x) = e^{1+x}$

(q) $y(x) = a^x + x^a$

(r) $f(x) = \sin(\pi x) + \cos(\frac{\pi}{2})e^{-x}$

(s) $f(x) = a^{5x}$

(t) $f(x) = 2^x + 2 \cdot 3^x$

(u) $y = \dfrac{3^x}{3} + \dfrac{33}{\sqrt{x}}$

(v) $f(t) = (\ln 3)^t$

(w) $f(x) = e^2 + x^e$

(x) $f(x) = \pi^x + x^\pi$

(y) $f(t) = e^{t+2}$

(z) $f(x) = x^{\pi^2} + \pi^{2x}$

Problem 4. Are the following statements true or false? Give an explanation for your answer.

(a) If $f(x)$ is increasing, then $f'(x)$ is increasing.

(b) There is no function such that $f'(x) = f(x)$ for all x besides the constant function $f(x) = 0$.

(c) There is no function such that $f'(x) = -f(x)$ for all x besides the constant function $f(x) = 0$.

(d) There is no function such that $f''(x) = -f(x)$ for all x besides the constant function $f(x) = 0$.

(e) If $f(x)$ is defined for all x, then $f'(x)$ is defined for all x.

Problem 5. *(Winter 2016 Exam 3)* For constants A and B, consider the function h defined by

$$h(t) = \begin{cases} (At)^2 - 48 & \text{if } t < 2 \\ Bt^3 & \text{if } t \geqslant 2. \end{cases}$$

Circle <u>all</u> pairs of values of A and B such that $h(t)$ is differentiable.

i. $A = 3, B = 3$

ii. $A = 6, B = 12$

iii. $A = -6, B = 12$

iv. $A = 0, B = 0$

v. $A = 18, B = 12$

vi. NONE OF THESE

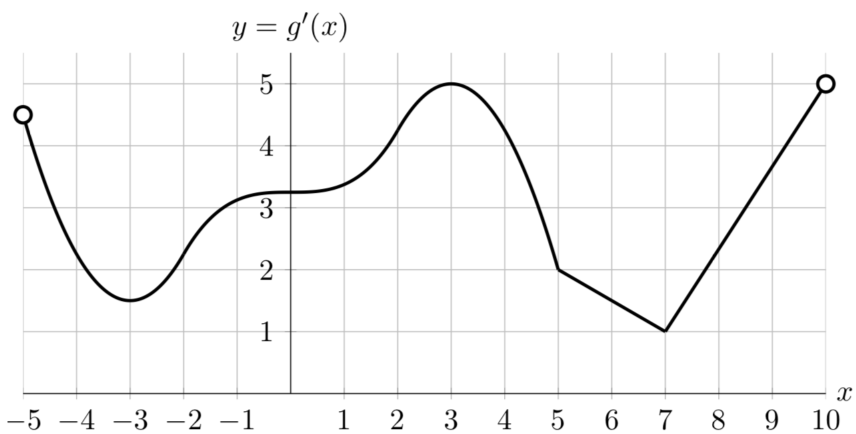
Problem 6. For what value(s) of a are $y = a^x$ and $y = x + 1$ tangent at $x = 0$?

Problem 7 (Winter 2018 Exam 2 Problem 6)**.** The function $P(t)$ is given by the equation

$$P(t) = \begin{cases} t + 4 & t < 2 \\ t^2 - 3t + 8 & 2 \leqslant t \leqslant 3 \\ \frac{1}{9}(t^3 + 44) & t > 3 \end{cases}$$

For which values of t is P(t) differentiable? Show all your work to justify your answer.

Problem 8 (Winter 2018 Exam 2 Problem 8)**.** The graph of the derivative $g'(x)$ of the function $g(x)$ with domain $-5 < x < 10$ is shown below.



The function $g'(x)$ has corners at $x = 5$ and $x = 7$, and it is linear on the intervals $(5, 7)$ and $(7, 10)$. If there is not enough information given to answer the question, write NEI. If the answer is none, write NONE.

(a) Estimate the interval(s) on which the function $g(x)$ is concave up.

(b) Estimate the values of x in $5 < x < 10$ for which $g''(x)$ is not defined.

(c) Estimate the interval(s) on which $g'''(x) > 0$. Recall that $g(x)$ is the derivative of $g(x)$.