Worksheet 16

Warm-up question

$$(f(x)g(x))' =$$

$$\left(\frac{f(x)}{g(x)}\right)' =$$

Problem 0. Using only the quotient rule and the derivatives of sine and cosine, find the derivative of the function $f(x) = \tan(x)$.

Problem 1. Find the derivative of the following functions:

i. $f(x) = x \cdot 2^x$

ii. $f(t) = \sin(5)(t^2 + 3)e^t$

iii. $f(w) = \dfrac{w^{3.2}}{5^w}$

iv. $f(t) = \dfrac{t - 3}{t + 3}$

v. $f(x) = 2t \cdot x \cdot e^t - \dfrac{1}{\sqrt{t}}$

vi. $f(y) = \dfrac{4}{3 + \sqrt{y}}$

vii. $f(z) = \dfrac{az + b}{cz + k}$

viii. $f(x) = (2 - 3x^2)(6x^e - 3\pi)$

ix. $f(x) = (3x^2 + \pi)(e^x - 4)$

Problem 2 (Fall 2017 Exam 2 Problem 1). Let g be a twice differentiable function defined on $-1 < x < 11$. Some values of $g(x)$, $g'(x)$ and $g''(x)$ are shown in the table below.

x	0	2	4	6	8	10
$g(x)$	-2	-1	3	4	5	6
$g'(x)$	0.5	2	?	5	1	2
$g''(x)$	2	1	5	-3	-1	0.5

(a) Let $k(x) = g(x)g'(x)$. Find the value of $g'(4)$ if $k'(4) = 15$.

(b) Let $r(x) = \frac{\sin(x)}{g(x)}$. Find $r'(0)$.

Problem 3. Consider the family of functions

$$f(x) = \frac{ax^b}{e^x}, \quad x \geqslant 0$$

where $a > 0$ and $b > 1$ are positive constants. Find the values of x for which the tangent lines of $y = f(x)$ are horizontal. Your answer will contain a and b.