Worksheet 17

Warm-up question

$(f(g(x)))' =$

Problem 0.

(a) Use the chain rule to differentiate (with respect to x) both sides of the identity $f\big(f^{-1}(x)\big) = x$.

(b) Having done that, solve for $\frac{d}{dx} f^{-1}(x)$.

(c) Now with this trick, find the derivatives (as functions of x) of the following functions..

(Group 1) $f(x) = \ln(x)$
(Group 2) $f(x) = \log_a(x)$
(Group 3) $f(x) = \arcsin(x)$
(Group 4) $f(x) = \arctan(x)$
(Group 5) $f(x) = \arccos(x)$

Problem 1. Exercises 1, 4, 20, 27, 33, 35, 38, 40, 47, 49, 50, 53, 56 in Section 3.4 of the book.

Problem 2. On what intervals is $\ln(x^2+1)$ concave up?

Problem 3 (Fall 2017 Exam 2 Problem 9)**.** Let A and B be two constants and

$$h(x) = \begin{cases} 2Bx + A\ln(x) & \text{if } 0 < x \leqslant 1 \\ \dfrac{4A}{x} + Bx - 1 & \text{if } 1 < x \leqslant 2 \end{cases}$$

Find all the values of A and B that make the function $h(x)$ differentiable on the interval $0 < x < 2$. If no such values exist, write none. Justify your answer.

Problem 4 (Winter 2018 Exam 2 Problem 1)**.** Some values of the twice differentiable function f(x) and of its first and second derivative are given by the following table:

x	0	1	2	4	5	6	7
$f(x)$	1			4	4.3	5	
$f'(x)$			8		0.25	0.6	2
$f''(x)$	4				0.1	0.2	

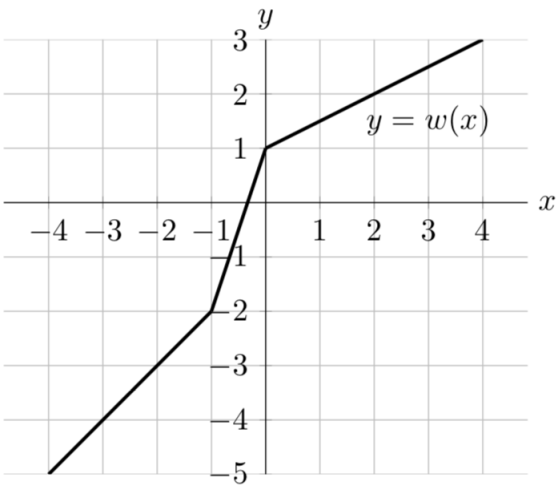
Suppose the function $f(x)$ is defined and invertible for $-\infty < x < \infty$. In the following questions, you will find some of the missing values using the information given. If there is not enough information given to answer the question, write "NEI". Show your work.

(a) The function $a(x) = \ln(1 + f(x))$ satisfies $a'(2) = 2$. Find $f(2)$.

(b) Let $b(x) = f(x)f'(x)$ and $b'(0) = 4$. Find $f'(0)$.

(c) Let $h(x) = f^{-1}(x)$. Find the value of $h'(5)$.

Problem 5 (Winter 2017 Exam 2 Problem 4)**.** A portion of the graph of the function $w(x)$ is shown below.



For each of the parts below, find the value of the given quantity. If there is not enough information provided to find the value, write *not enough info*. If the value does not exist, write *does not exist*. You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown. All your answers must be in exact form.

(a) Let $k(x) = w^{-1}(x)$. Find $k'(-1.5)$.

(b) Let $h(u) = \ln(3w(u))$. Find the value of $h'(1)$.

(c) Let $n(x) = \frac{w(x)}{1-x^2}$. Find $n'(-2)$.

(d) Let $s(x)$ be the exponential function $s(x) = 4^{w(x)}$. Find $s'(2)$.

(e) Let $p(x) = xw^{-1}(x)$. Find $p'(-1)$.

Problem 6 (Fall 2016 Exam 2 Problem 1)**.** The table below gives several values for the function f and its derivative f'. You may assume that f is invertible and differentiable.

w	-2	-1	0	1	2
$f(w)$	1	0	-2	-3	-5
$f'(w)$	-3	-1.5	-0.5	0	-4

For each of the parts below, find the exact value of the given quantity. If there is not enough information provided to find the value, write *not enough info*. If the value does not exist, write *does not exist*. You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

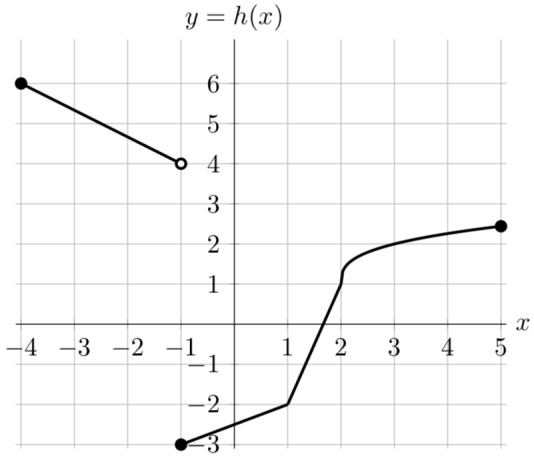
(a) Let $h(w) = \frac{f(w)}{6+w}$. Find $h'(2)$.

(b) Let $k(w) = 3^{f(2w)}$. Find $k'(1)$.

(c) Let $p(w) = f(f(-w+1))$. Find $p'(1)$.

(d) Let $r(w) = w(f(w))^2$. Find $r'(2)$.

Problem 7 (Fall 2017 Exam 2 Problem 4 (adapted))**.** Consider the graph of $h(x)$ below. Note that h is linear on the intervals $[-4,-1)$, $[-1,1]$, and $[1,2]$, differentiable on $(2,5)$, and has a sharp corner at $x=2$.



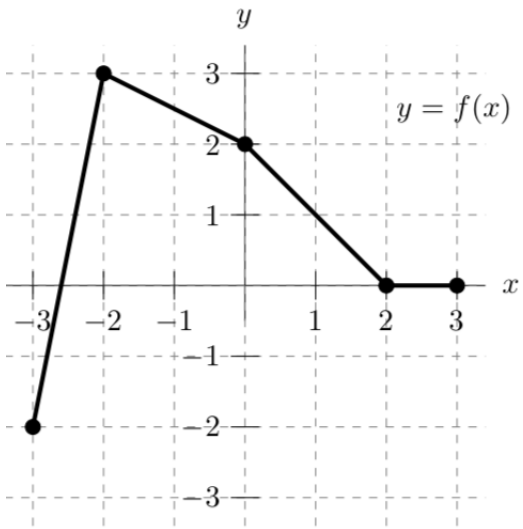
Find the exact value of the following expressions. If there is not enough information provided to find the value, write NI. If the value does not exist, write DNE. Show all your work.

(a) Let $g(x)=xh(x)$. Find $g'(-2)$.

(b) Let $p(x)=h^{-1}(x)$. Find $p'(0)$.

(c) Find $h'(-1)$.

Problem 8 (Winter 2016 Exam 2 Problem 2)**.** Let f be the piecewise linear function with graph shown below.



The table below gives several values of a differentiable function g and its derivative g'. Assume that both $g(x)$ and $g'(x)$ are invertible.

x	-2	-1	0	2	5
$g(x)$	21	11	5	-1	-3
$g'(x)$	-12	-8	-4	-2	-0.4

For each of parts below, find the value of the given quantity. If there is not enough information provided to find the value, write *not enough info*. If the value does not exist, write *does not exist*.

(a) Let $j(x)=e^{g(x)}$. Find $j'(2)$.

(b) Let $k(x)=f(x)f(x+2)$. Find $k'(1)$.

(c) Let $h(x)=3f(x)+g(x)$. Find $h'(-2)$.

(d) Find $(g^{-1})'(2)$.

(e) Let $m(x)=g(f(g(x)))$. Find $m'(2)$.

(f) Let $l(x)=\frac{f(x)}{2g(x)}$. Find $l'(-1)$.

Problem 9 (Fall 2016 Exam 2 Problem 8)**.** Pepukai is studying the effect of the availability of water on the fruit productivity of Michigan apple trees. She observes that Michigan apple trees produce very few apples if they have too little water. She determines a function $p(w)$ that models the total weight, in pounds, of all the apples that an average Michigan apple tree produces in a season when it is watered with w gallons of water every week. The domain of p is $[5, 40]$. Some values of the function p and its derivative p' are shown in the table below.

w	10	15	20	25	30
$p(w)$	25	96	118	129	135
$p'(w)$	96	13	4	2	1

The function p is invertible and the functions p, p', and p^{-1} are all differentiable. Furthermore, the function p' is always decreasing.

(a) Find $(p^{-1})'(96)$.

(b) Circle the one statement that is best supported by the equation

$$(p^{-1})'(10) = 0.01$$

(i) To increase the total weight of apples produced in a season by an average Michigan apple tree from 10 pounds to 11 pounds, the tree should be watered with about 0.01 additional gallons of water every week.

(ii) If an average Michigan apple tree produces 10 pounds of apples in a season, watering the tree with 1 extra gallon every week increases the total weight of apples produced by the tree in a season by about 0.01 pounds.

(iii) If the amount of water that an average Michigan apple tree is watered with increases from 10 gallons every week to 10.1 gallons every week, the total weight of apples produced by the tree in a season increases by about 10 pounds.

(iv) If the amount of water that an average Michigan apple tree is watered with increases from 10 gallons every week to 10.1 gallons every week, the total weight of apples produced by the tree in a season increases by about 0.001 pounds.