Worksheet 18

Problem 1 (Winter 2018 Exam 2 Problem 2)**.** Find $\dfrac{dy}{dx}$ for the implicit function given by

$$2^{x+y} + \sin(x)\cos(y) = 5 - x.$$

Problem 2 (Exam 2 Fall 2016)**.** Let a and b be constants. Consider the curve $\mathcal{C}$ defined by the equation

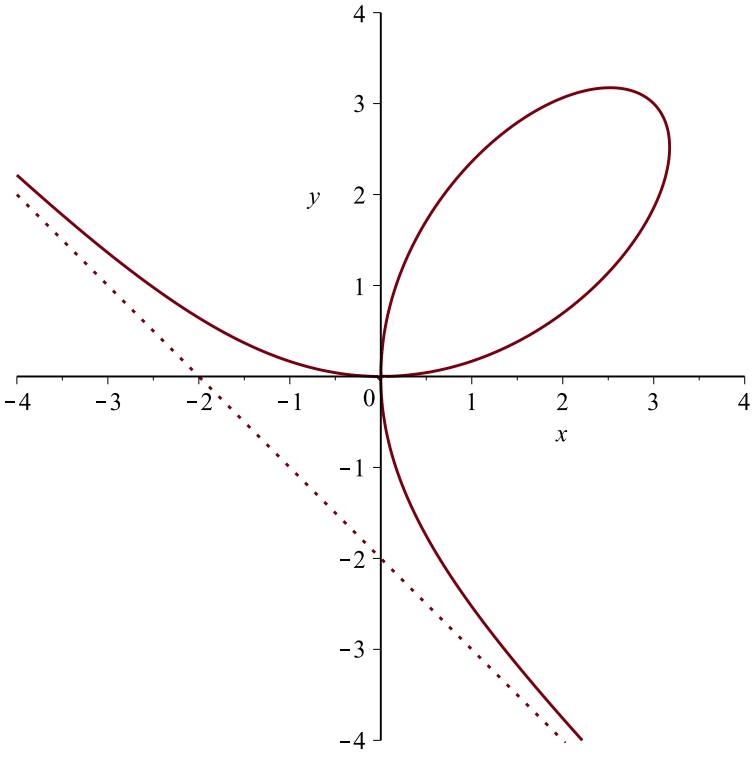
$$\cos(ax) + by\ln(x) = 3 + y^3.$$

Find the formula for $\frac{dy}{dx}$ in terms of x and y. The constants a and b may appear in your answer.

Problem 3 (Descartes' folium)**.** The folium of Descartes (a.k.a. Descartes' leaf) was first discovered in 1638 and is defined as the curve

$$x^3 + y^3 = 6xy.$$

The curve forms a loop in the first quadrant, and it is symmetric about $y = x$ with asymptote $x + y + 2 = 0$:

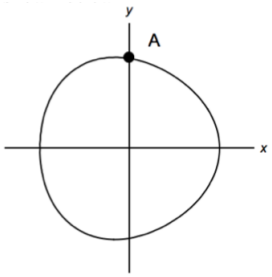


(a) Show that the point $(x, y) = (3, 3)$ lies on the curve.

(b) Find the equation of the tangent line to the curve at the point $(x, y) = (3, 3)$.

(c) For what value(s) of x (if any) will the tangent line to this curve be horizontal?

Problem 4 (Fall 2017 Exam 2 Problem 8)**.** Let $\mathcal{C}$ be the curve given by the equation $81-(x^2+y^2)^2 = 2xy^2$. The graph of $\mathcal{C}$ is shown below.



(a) Find the coordinates (x, y) of the point A.

(b) Find $\dfrac{dy}{dx}$.

(c) Find the equation of tangent line $L(x)$ to the graph of $\mathcal{C}$ at A.

Show your computations step by step.

Problem 5.

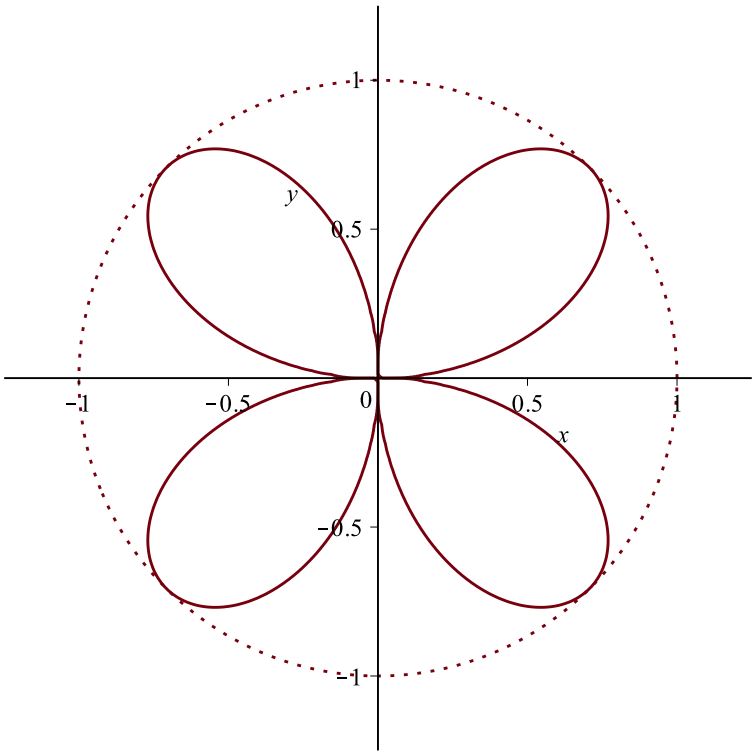
(a) Find $\dfrac{dy}{dx}$ and $\dfrac{dx}{dy}$ given that $x^2 + y^2 - 4x + 7y = 15$.

(b) Under what conditions on x and/or y is the tangent line to this curve horizontal? Vertical?

Problem 6 (A rose curve)**.** A *quadrifolium* is a 4-petaled rose curve given by

$$\left(x^2 + y^2\right)^3 = 4x^2y^2$$

and which is shown below as the solid curve. It lies inside the unit circle $x^2 + y^2 = 1$.[1]



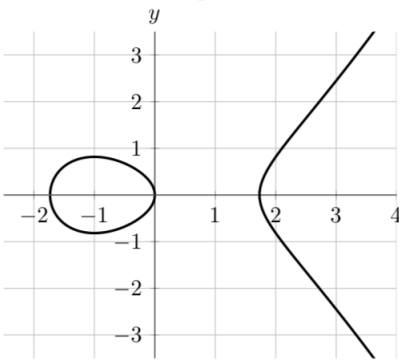
(a) Show that the point $(x, y) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ lies on the quadrifolium.

(b) Find the equation of the tangent line to the quadrifolium at $(x, y) = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

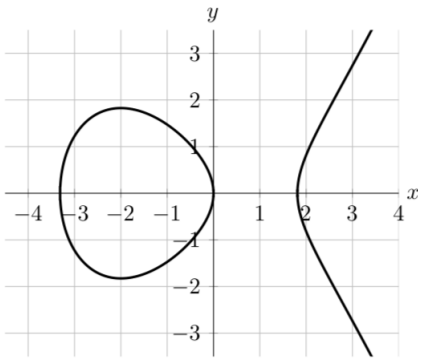
[1] Fun fact: quite surprisingly the area of the quadrifolium equals $\frac{\pi}{2}$!

Problem 7 (Winter 2018 Exam 2 Problem 2)**.** Each the following is the graph of an implicit function.

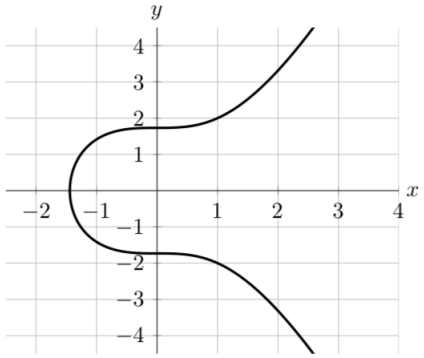
Graph I



Graph II



Graph III



Match each of the graphs above to the formula below that gives the slope at each point on the graph.

(a) $\dfrac{dy}{dx} = \dfrac{3x^2}{2y}$

(b) $\dfrac{dy}{dx} = \dfrac{(x-1)(x+2)}{2y}$

(c) $\dfrac{dy}{dx} = \dfrac{x^2-1}{2y}$

(d) $\dfrac{dy}{dx} = \dfrac{(y-1)(y+2)}{2x}$