Worksheet 19

Warm-up questions

The linear approximation or local linearization of $f(x)$ at $x = a$ is given by $L(x) =$

The quadratic approximation of $f(x)$ at $x = a$ is given by $Q(x) =$

Problem 1 (Fall 2016 Exam 2 Problem 11)**.** Let $h(x) = x^x$. For this problem, it may be helpful to know the following formulas:

$$h'(x) = x^x(\ln(x) + 1) \qquad \text{and} \qquad h''(x) = x^x\left(\frac{1}{x} + (\ln(x) + 1)^2\right)$$

(a) Write a formula for $p(x)$, the local linearization of $h(x)$ near $x = 1$.

(b) Write a formula for $u(x)$, the quadratic approximation of $h(x)$ at $x = 1$.

Problem 2 (Winter 2018 Exam 2 Problem 1)**.** The quadratic approximation $Q(x)$ of the function $f(x)$ at $x = 1$ is $Q(X) = \frac{1}{2}x + \frac{3}{2}$. Find $f(1)$, $f'(1)$ and $f''(1)$.

Problem 3. Let $R(x)$ be a polynomial whose first and second derivatives are given below.

$$R'(x) = (x-1)^7(x+2)^4 \text{ and } R''(x) = (11x+10)(x-1)^6(x+2)^3$$

Find the quadratic approximation $G(x)$ of $R(x)$ at the point $(-1, 5)$ on the graph of $R(x)$. Show all your work.

Problem 4 (Fall 2017 Exam 2 Problem 1 b.)**.** Let g be a twice differentiable function defined on $-1 < x < 11$. Some values of $g(x)$, $g'(x)$ and $g''(x)$ are shown in the table below.

x	0	2	4	6	8	10
$g(x)$	-2	-1	3	4	5	6
$g'(x)$	0.5	2	?	5	1	2
$g''(x)$	2	1	5	-3	-1	0.5

Let $j(x) = g(14 - 4x)$.

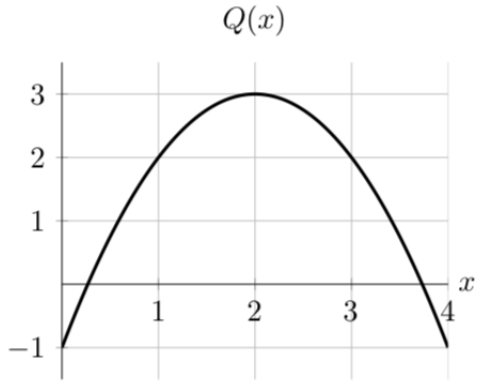
(a) Use the values from the table to find a formula for $L(x)$, the linear approximation to $j(x)$ at $x = 2$.

(b) Find an approximate value for $j(2.25)$ using your formula for $L(x)$.

(c) Is your approximation in part (b) an overestimate or an underestimate? Circle your answer and give a justification of your answer.

Overestimate Underestimate Not enough information

Problem 5 (Winter 2018 Final Exam Problem 9)**.** Let $Q(x) = -(x-2)^2 + 3$ be the quadratic approximation of the function $y = f(x)$ at $x = 3$. A part of the graph of $Q(x)$ is shown below.



(a) If possible, find the following quantities exactly. If there is not enough information, write *not enough information*.

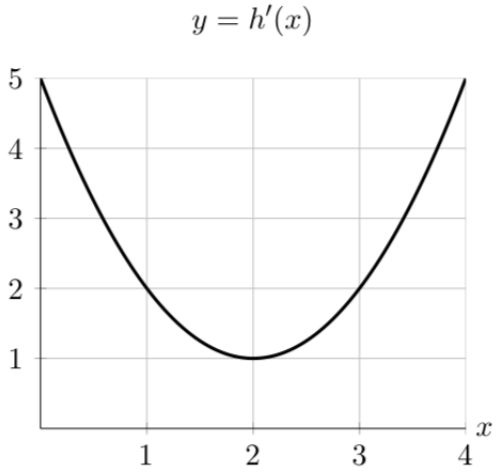
(1) $f''(3)$ (3) $f(0)$ (5) $Q'''(3)$

(2) $f'''(3)$ (4) $Q''(3)$ (6) $Q(0)$

(b) Assume that the function $f(x)$ is invertible and let $g(y) = f^{-1}(y)$ be its inverse. Given that f(3) = 2, find the linear approximation $L(y)$ of $g(y)$ at $y = 2$. Your answer should not include the letters f or g. Show all your work.

(c) Use the linear approximation $L(y)$ to approximate a solution to the equation $f(x) = 1.7$.

Problem 6 (Winter 2018 Exam 2 Problem 9)**.** Below is the graph of $h'(x)$.



(a) Find a formula for the tangent line approximation $L(x)$ to the function $h(x)$ near $x = 2$ if the point $(2, -3)$ lies on the graph of $y = h(x)$. Your answer should not include the letter h.

(b) Use the tangent line approximation to $h(x)$ near $x = 2$ to approximate the value of $h(1.6)$.

(c) Is your approximation in part (b) an overestimate, an underestimate or is there not enough information to determine that?