Worksheet 20

Warm-up questions

What are the hypothesis of the mean value theorem?

What is the conclusion of the mean value theorem?

Problem 0. Give an example of a function f that is...

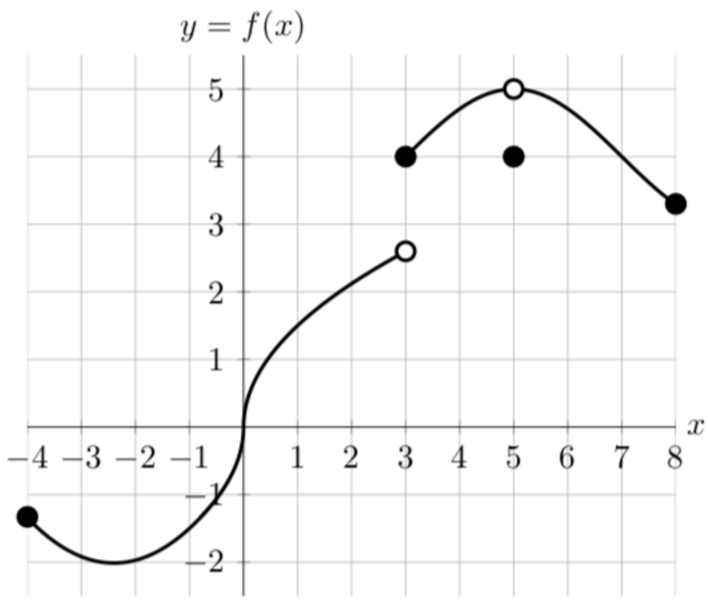
(a) continuous on the interval $[-1, 1]$ but that does not satisfy the hypothesis of the Mean Value Theorem on that interval, and for which the conclusion of the Mean Value Theorem is false.

(b) continuous on the interval $[-1, 1]$ but that does not satisfy the hypothesis of the Mean Value Theorem on that interval, and for which the conclusion of the Mean Value Theorem is true.

(c) differentiable on the interval $(0, 2)$ but that does not satisfy the hypothesis of the Mean Value Theorem on $[0, 2]$.

Problem 1 (Winter 2018 Exam 2 Problem 5)**.** The graph of the function f with domain $-4 \leqslant x \leqslant 8$ is shown below. The function $f(x)$ satisfies:

- $f(x) = 1.5x^{\frac{1}{3}}$ for $-1 < x < 1$, and
- $f(x) = 4 + \sin\left(\frac{\pi}{4}(x-3)\right)$ for $3 \leqslant x < 5$ and $5 < x \leqslant 8$.



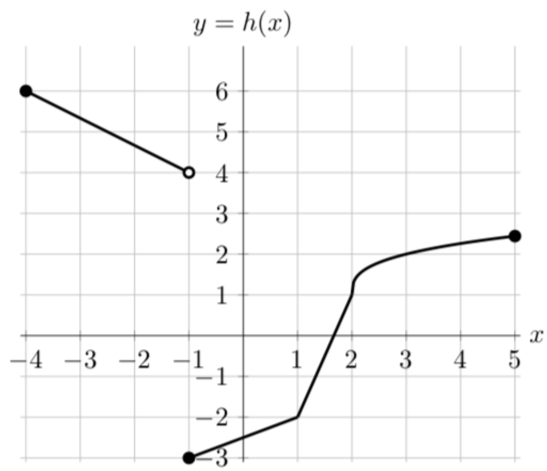
(a) On which of the following intervals is the conclusion of the Mean Value Theorem true?

$[-4, 0]$ $[0, 5]$ $[1, 3]$ $[3, 7]$ none of these

(b) On which of the following intervals are the hypothesis of the Mean Value Theorem true?

$[-4, 0]$ $[0, 5]$ $[1, 3]$ $[3, 7]$ none of these

Problem 2 (Fall 2017 Exam 2 Problem 4). Consider the graph of $h(x)$ below. Note that h is linear on the intervals $[-4,-1)$, $[-1,1]$ and $[1,2]$, differentiable on $(2,5)$, and has a sharp corner at $x=2$.



(a) On which of the following intervals is the conclusion of the Mean Value Theorem true?

$[-4,-1]$ $[-2,-1]$ $[0,4]$ $[2,5]$ none of these

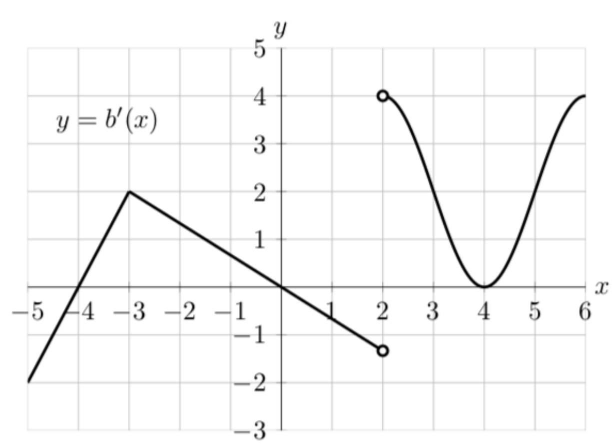
(b) On which of the following intervals are the hypothesis of the Mean Value Theorem true?

$[-4,-1]$ $[-2,-1]$ $[0,4]$ $[2,5]$ none of these

(c) For which values given below is the function $m(x) = h(h(x))$ not differentiable? Circle all that apply.

$x=-3$ $x=-1$ $x=2$ $x=3$ $x=4$ none of these

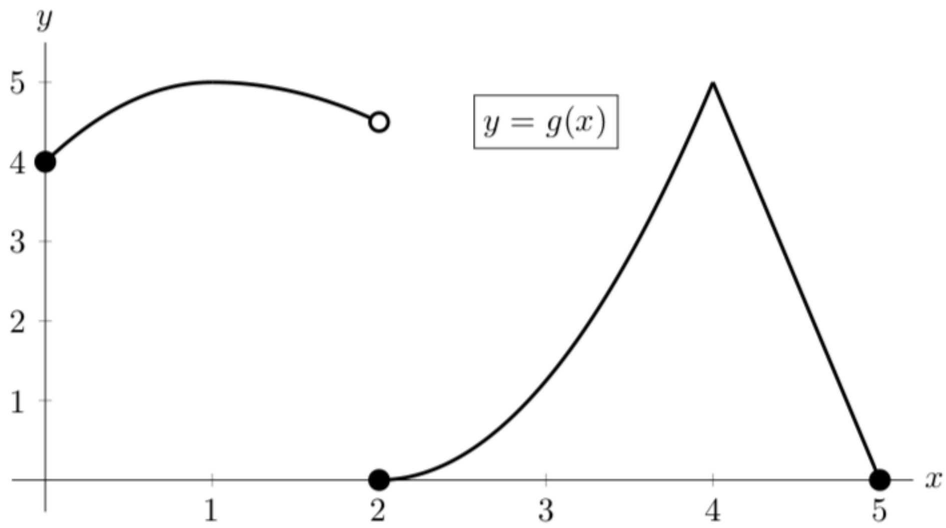
Problem 3 (Winter 2017 Exam 2 Problem 1). The graph of a portion of the derivative of $b(x)$ is shown below. Assume that $b(x)$ is defined and continuous on $[-5,6]$.



On which of the following intervals are the hypothesis of the Mean Value Theorem true?

$[-4,-2]$ $[-2,2]$ $[1,4]$ $[-5,6]$ none of these

Problem 4 (Fall 2016 Exam 2 Problem 6)**.** The entire graph of a function $g(x)$ is shown below. Note that the graph of $g(x)$ has a horizontal tangent line at $x = 1$ and a sharp corner at $x = 4$.



(a) Let $L(x)$ be the local linearization of $g(x)$ near $x = 3$. Circle all the statements that are true.

(1) $L(3) > g(3)$	(7) $L(2.5) > g(2.5)$	(13) $L(0) > g(0)$
(2) $L(3) = g(3)$	(8) $L(2.5) = g(2.5)$	(14) $L(0) = g(0)$
(3) $L(3) < g(3)$	(9) $L(2.5) < g(2.5)$	(15) $L(0) < g(0)$
(4) $L'(3) > g'(3)$	(10) $L'(2.5) > g'(2.5)$	(16) $L(5) > g(5)$
(5) $L'(3) = g'(3)$	(11) $L'(2.5) = g'(2.5)$	(17) $L(5) = g(5)$
(6) $L'(3) < g'(3)$	(12) $L'(2.5) < g'(2.5)$	(18) $L(5) < g(5)$

(19) None of these

(b) On which of the following intervals does g satisfy the hypothesis of the Mean Value Theorem?

$[0, 2]$ $[0, 4]$ $[3, 5]$ $[4, 5]$ none of these

(c) On which of the following intervals does g satisfy the conclusion of the Mean Value Theorem?

$[0, 2]$ $[0, 4]$ $[3, 5]$ $[4, 5]$ none of these

Problem 5 (Winter 2016 Exam 2 Problem 4)**.** Let $h(x)$ be a twice differentiable function defined for all real numbers x. (So h is differentiable and its derivative h' is also differentiable.) Some values of the derivative of h are given in the table below.

x	-8	-6	-4	-2	0	2	4	6	8
$h'(x)$	3	7	0	-3	-5	-4	0	-2	6

(a) Circle all the intervals which must contain a number c such that $h''(c) = 2$.

$-8 < x < -6$ $\quad -4 < x < -2$ $\quad -2 < x < 0$ $\quad 6 < x < 8$

(b) Suppose that $h''(x) < 0$ for $x < -8$ and $h(-8) = 7$. Circle all the numbers below which could equal the value of $h(-10)$.

-2 $\quad -1$ $\quad 0$ $\quad 1$ $\quad 2$ $\quad$ None of these