Worksheet 21

Warm-up questions

A critical point of f is a point a such that

Critical points can be

When do we apply the first derivative test?

The first derivative test says that f has a local minimum at a if

The first derivative test says that f has a local maximum at a if

When do we apply the second derivative test?

The second derivative test says that f has a local minimum at a if

The second derivative test says that f has a local maximum at a if

The second derivative test says nothing useful if

How can we detect an inflection point?

Problem 0. Give an example of

(a) a function f and a critical point a for f such that f *does not* have a local maximum or a local minimum at $x = a$.

(b) a function f and a value of a such that $f'(a) = f''(a) = 0$ but f *does not* have an inflection point at a.

Problem 1 (Winter 2018 Exam 2 Problem 4)**.** In the following question, use calculus to justify your answers and show enough evidence to demonstrate that you have found them all. Determine your answers algebraically.

(a) Let $f(x)$ be a continuous function defined for all real numbers with derivative given by

$$f'(x) = \frac{(2x+1)(x-2)^2}{(x+3)^{\frac{1}{3}}}.$$

Find the x-coordinate(s) of the local maximum(s) and local minimum(s) of the function $f(x)$. Write *none* if the function has no local maximum(s) and/or local minimum(s).

(b) Let $g(x)$ be a continuous function defined for all real numbers with second derivative given by

$$g''(x) = (2^x - 4)(x^2 - 4).$$

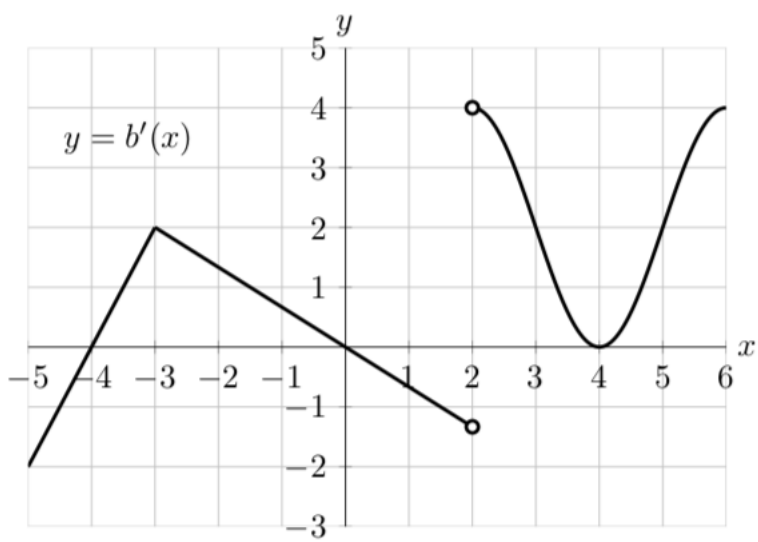
Find the x-coordinates of the inflection points of the function $g(x)$. Write *none* if the function has no inflection points.

Problem 2 (Fall 2017 Final Exam Problem 7)**.** Let A and B be positive constants and $f(x) = \frac{A(x^2-B)}{\sqrt{x-3}}$ for all $x > 3$. Note that

$$f'(x) = \frac{A(x^2 - 12x + B)}{2(x-3)^{\frac{3}{2}}} \qquad \text{and} \qquad f''(x) = \frac{2A(x^2 - 8x + 24 - B)}{4(x-3)^{\frac{5}{2}}}.$$

Find all the values A and B such that f has an inflection point at $(8, 2)$. Use calculus to justify that $(8, 2)$ is an inflection point. If there are no such values, write *none*.

Problem 3 (Winter 2017 Exam 2 Problem 1)**.** The graph of a portion of the derivative of $b(x)$ is shown below. Assume that $b(x)$ is defined and continuous on $[-5, 6]$.



(a) At which of the following values of x does $b(x)$ appear to have a critical point?

$x = -4$ $\quad$ $x = -3$ $\quad$ $x = 2$ $\quad$ $x = 3$ $\quad$ none of these

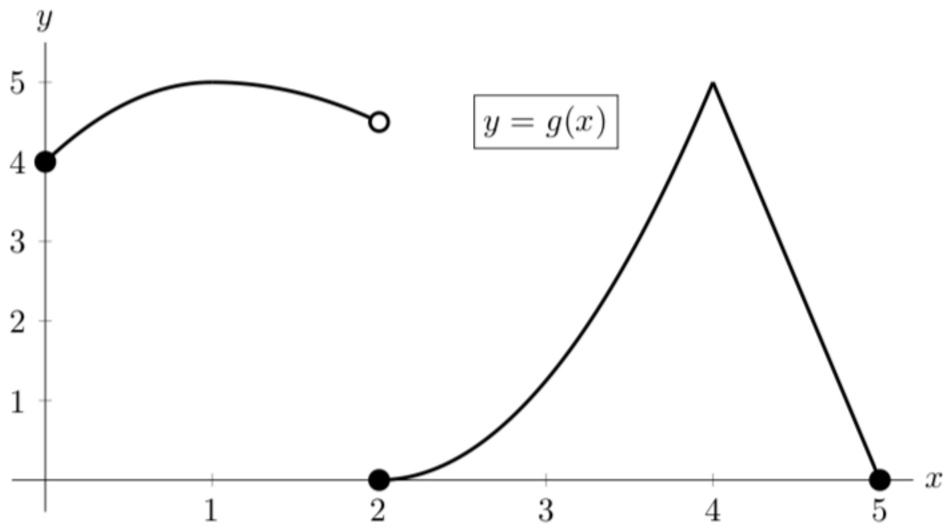
(b) At which of the following values of x does $b(x)$ attain a local minimum?

$x = -4$ $\quad$ $x = 2$ $\quad$ $x = 3$ $\quad$ $x = 5$ $\quad$ none of these

(c) At which of the following values of x does $b(x)$ appear to have an inflection point?

$x = -3$ $\quad$ $x = 2$ $\quad$ $x = 3$ $\quad$ $x = 5$ $\quad$ none of these

Problem 4 (Fall 2016 Exam 2 Problem 6). The entire graph of a function $g(x)$ is shown below. Note that the graph of $g(x)$ has a horizontal tangent line at $x = 1$ and a sharp corner at $x = 4$.



(a) At which of the following values of x does $g(x)$ appear to have a critical point?

$x = 1$ $\quad x = 2$ $\quad x = 3$ $\quad x = 4$ $\quad$ none of these

(b) At which of the following values of x does $g(x)$ attain a local maximum?

$x = 1$ $\quad x = 2$ $\quad x = 3$ $\quad x = 4$ $\quad$ none of these

Problem 5 (Winter 2016 Exam 2 Problem 4). Let $h(x)$ be a twice differentiable function defined for all real numbers x. (So h is differentiable and its derivative h' is also differentiable.) Some values of the derivative of h are given in the table below.

x	-8	-6	-4	-2	0	2	4	6	8
$h'(x)$	3	7	0	-3	-5	-4	0	-2	6

(a) Circle all the intervals below in which $h(x)$ must have a critical point.

$-8 < x < -6$ $\quad -6 < x < 2$ $\quad -2 < x < 2$ $\quad 2 < x < 6$ $\quad 6 < x < 8$ $\quad$ none of these

(b) Circle all the intervals below in which $h(x)$ must have a local extremum (i.e. a local maximum or a local minimum).

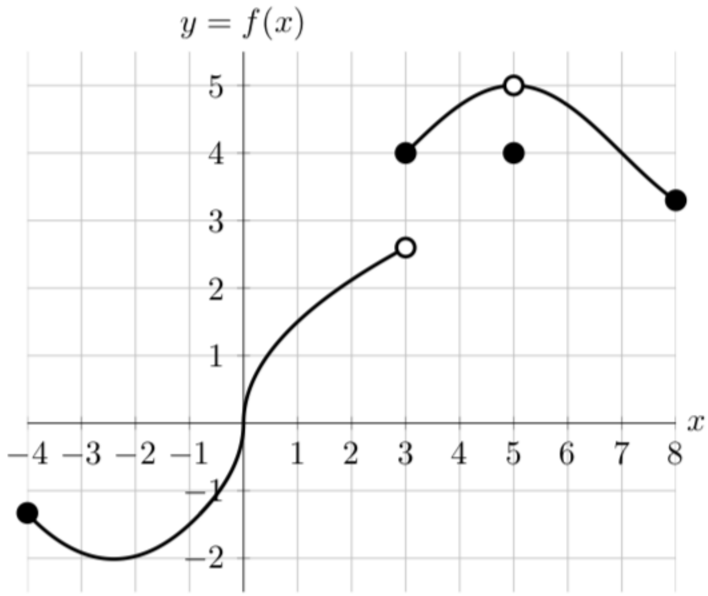
$-8 < x < -6$ $\quad -6 < x < -2$ $\quad -2 < x < 2$ $\quad 2 < x < 6$ $\quad 6 < x < 8$ $\quad$ none of these

(c) Circle all the intervals below in which $h(x)$ must have an inflection point.

$-8 < x < -4$ $\quad -4 < x < 0$ $\quad 0 < x < 4$ $\quad 2 < x < 6$ $\quad 4 < x < 8$ $\quad$ none of these

Problem 6 (Winter 2018 Exam 2 Problem 5)**.** The graph of the function f with domain $-4 \leqslant x \leqslant 8$ is shown below. The function $f(x)$ satisfies:

- $f(x) = 1.5x^{\frac{1}{3}}$ for $-1 < x < 1$, and
- $f(x) = 4 + \sin\left(\frac{\pi}{4}(x-3)\right)$ for $3 \leqslant x < 5$ and $5 < x \leqslant 8$.



(a) Estimate the x-coordinate(s) of all the local minimum(s) of $f(x)$ in $-4 < x < 8$. Write *none* if $f(x)$ does not have any local minima.

(b) Find the value(s) of b in $-4 < x < 8$ for which the limit $\lim_{h \to 0} \frac{f(b+h) - f(b)}{h}$ does not exist. Write *none* if there are no such values of b.

(c) Estimate the x-coordinate(s) of all critical points of $f(x)$ in $-4 < x < 8$. Write *none* if $f(x)$ does not have any critical points.