

## Worksheet 25

## Warm-up questions

The **cost function**  $C(q)$  gives the cost of producing a quantity  $q$  of a certain good.

The **revenue function**  $R(q)$  gives the revenue received from selling a quantity  $q$  of some good.

The **profit**  $\pi(q) =$  \_\_\_\_\_ gives the total profit from producing and selling  $q$  of that good.

To decide whether a company's profit would increase or decrease if the company increased or decreased production of a certain good, we might look at the marginal cost and marginal revenue:

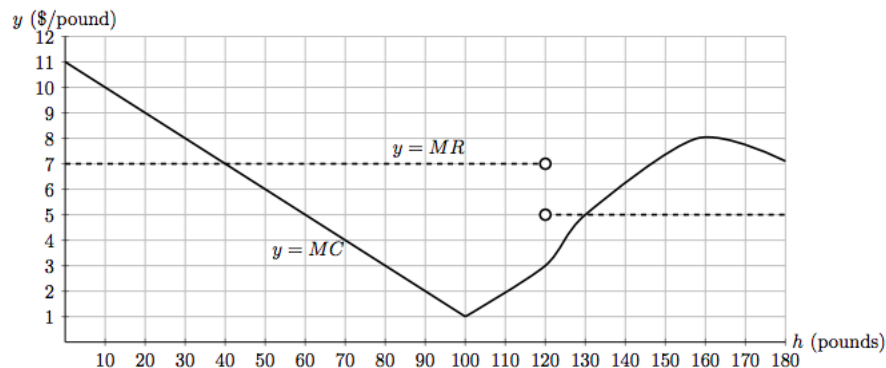
The **marginal cost** is given by  $MC(q) =$  \_\_\_\_\_  $\approx$  \_\_\_\_\_

The **marginal revenue** is given by  $MR(q) =$  \_\_\_\_\_  $\approx$  \_\_\_\_\_

When can maximum profit occur?

How do we identify the fixed cost of producing a certain good?

**Problem 1** (Winter 2017 Final Exam Problem 10). The Happy Hives Bee Farm sells honey. The graph below shows marginal revenue  $MR$  (dashed) and marginal cost  $MC$  (solid), in dollars per pound, where  $h$  is the number of pounds of honey.



- (a) Use the graph to estimate the answers to the following questions. If an answer can't be found with the information given, write *NEI*.
- For what value(s) of  $h$  in the interval  $[0, 180]$  is the cost function  $C$  minimized?
  - For what value(s) of  $h$  in the interval  $[0, 180]$  is  $MC$  minimized?
  - For what value(s) of  $h$  in the interval  $[0, 180]$  is profit maximized?
  - What are the fixed costs of the farm?
  - For what values of  $h$  in the interval  $[0, 180]$  is the profit function concave up?

**Warning:** we do not know how to answer the following question yet, but we will soon!

- (b) The farm currently sells 20 pounds of honey but is thinking of increasing to 80 pounds of honey. Will this increase or decrease profit? By approximately how much will the profit change?

## How do you like them apples

**Problem 2** (Fall 2016 Final Exam Problem 10). Yukiko has a small orchard where she grows Michigan apples. After careful study last season, Yukiko found that the total cost, in dollars, of producing  $a$  bushels of apples can be modeled by

$$C(a) = -25500 + 26000e^{0.002a} \quad \text{for } 0 \leq a \leq 320.$$

Qabil has promised to buy up to 100 bushels of apples for his famous apple ice cream. If Yukiko has any remaining apples, she has an agreement to sell them to Xanthippe's cider mill at a reduced price. Let  $R(a)$  be the revenue generated from selling  $a$  bushels of apples. Then

$$R(a) = \begin{cases} 70a & \text{for } 0 \leq a < 100 \\ 200 + 50a & \text{for } 100 < a \leq 320 \end{cases}$$

- How much will Xanthippe's cider mill pay per bushel?
- What is Yukiko's fixed cost?
- For what quantities of bushels of apples sold would Yukiko's marginal revenue equal her marginal cost? Write none if appropriate.
- Assuming Yukiko can produce up to 320 bushels of apples, how many bushels should she produce in order to maximize her profit, and what would that maximum profit be? You must use calculus to find and justify your answer. Make sure to provide enough evidence to justify your answer fully.

**Problem 3** (Fall 2017 Final Exam Problem 2). Jane has a company that produces a protein powder for an energy shake. The cost, in dollars, of producing  $m$  pounds of protein powder is given by the function

$$C(m) = \begin{cases} \frac{1}{4}(m+2)^2 + 8 & \text{for } 0 \leq m < 16 \\ 2m + 57 & \text{for } 16 \leq m \leq 30 \end{cases}$$

The revenue, in dollars, of selling  $m$  pounds of protein powder is given by

$$R(m) = 5m.$$

- What is the price, in dollars, at which Jane sells each pound of the protein powder?
- What is the fixed cost, in dollars, of producing Jane's protein powder?
- Find all values of  $16 \leq m \leq 30$  for which Jane's profit is positive.
- Find all the values of  $0 \leq m \leq 30$  where the marginal cost is equal to the marginal revenue for the protein powder. Show all your work to justify your answer.
- What is the maximum profit that Jane can make if she sells at most 30 pounds of protein powder? Use calculus to find and justify your answer, and make sure to provide enough evidence to fully justify your answer.

**Problem 4** (Winter 2016 Final Exam Problem 5). Reggie is starting a fruit punch company. He has determined that the total cost, in dollars, for him to produce  $q$  gallons of fruit punch can be modeled by

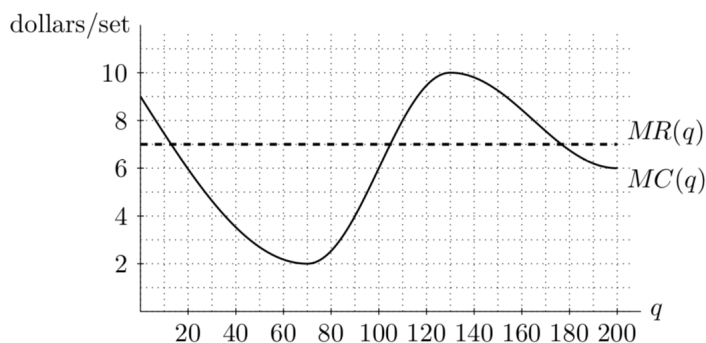
$$C(q) = 100 + q + 25e^{q/100}.$$

Reggie can sell up to 100 gallons to Chris at a price of \$4 per gallon, and he can sell the rest to Alice at a price of \$3 per gallon. Assume that Reggie sells all of the fruit punch that he produces.

Note: Assume that the quantities of fruit punch produced and sold do not have to be whole numbers of gallons. (For example, Reggie could produce exactly  $50\sqrt{2}$  gallons of fruit punch and sell all of these to Chris, who would pay a total of  $200\sqrt{2}$  dollars for them.)

- For what quantities of fruit punch sold would Reggie's marginal revenue equal his marginal cost?
- Assuming that Reggie can produce at most 200 gallons of fruit punch, how much fruit punch should he produce in order to maximize his profit, and what would that maximum profit be? You must use calculus to find and justify your answer. Be sure to provide enough evidence to justify your answer fully.

**Problem 5** (Fall 2013 Final Exam problem 2). Link has started a business selling winter clothes for cats. Among his most successful products are his new kitten mittens. He is currently selling his mittens for \$7 per set. Below is a graph of Link's marginal cost  $MC(q)$  and marginal revenue  $MR(q)$ , in dollars per set of mittens, if he makes  $q$  sets of mittens this winter. Due to a shortage of yarn, Link can make a maximum of 200 sets of mittens this winter. In order to start making mittens, Link must spend \$40 on knitting supplies (in other words, it costs \$40 to make 0 sets of mittens).



- Approximately how many sets of mittens should Link make this winter in order to maximize his profit?

**Warning:** we do not know how to answer the following question yet, but we will soon!

- If the price per set is raised to \$9, approximately how many sets of mittens should Link make in order to maximize his profit?
- Link makes a deal with a store that would like to buy his cat hats. If the store buys up to 50 hats, then each one will cost \$10. If the store buys more than 50 hats, then Link will reduce the price of the entire order by \$0.05 per hat for every additional hat over 50. (For example, if the store buys 52 hats, they will pay \$9.90 per hat.) Write a formula for a function  $L(q)$  which gives Link's revenue if he sells  $q$  hats to the store.