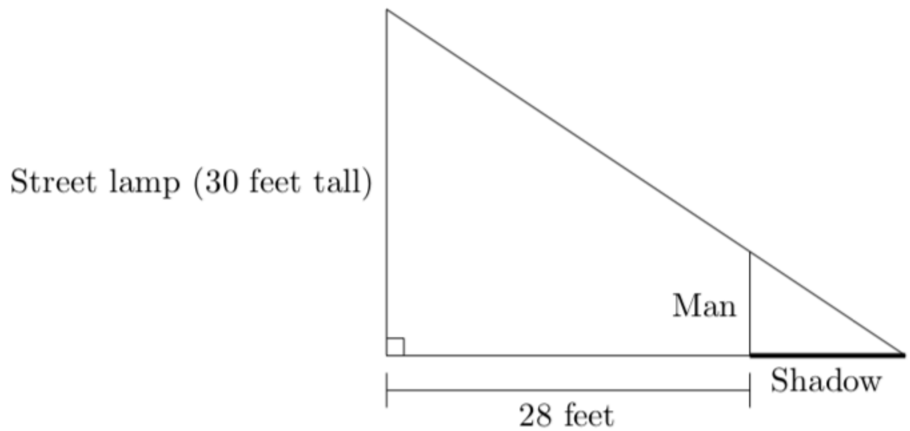
Worksheet 26

Problem 1 (Winter 2017 Final Exam problem 5). A cylindrical bar of radius R and length L (both in meters) is put into an oven. As the bar gains temperature, its radius decreases at a constant rate of 0.05 meters per hour and its length increases at a constant rate of 0.12 meters per hour. Fifteen minutes after the bar was put into the oven, its radius and length are 0.4 and 3 meters respectively. At what rate is the volume of the bar changing at that point? Be sure to include units.

Problem 2 (Winter 2016 Final Exam Problem 3). A man, who is 28 feet away from a 30 foot tall street lamp, is sinking into quicksand. (See diagram below.) At the moment when 6 feet of him are above the ground, his height above the ground is shrinking at a rate of 2 feet/second.



(a) How long will the man's shadow (shown in bold in the diagram above) be at the moment when 6 feet of him are above the ground?

(b) At what rate is the length of the man's shadow changing at the moment 6 feet of him are above the ground? Is his shadow growing or shrinking at that moment?

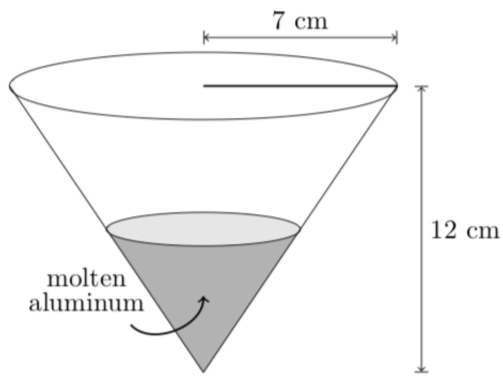
Problem 3 (Fall 2014 Final Exam Problem 5). Tommy and Gina were friends in high school but then went to college in different parts of the country. They thought they were going to see each other in Springfield over the December break, but their schedules didn't match up. In fact, it turns out that Tommy is leaving on the same day that Gina is arriving. Shortly before Gina's train arrives in Springfield, she sends a text to Tommy to see where he is, and Tommy sends a text response to say that, sadly, his train has already left. At the moment Tommy sends his text, he is 20 miles due east of the center of the train station and moving east at 30 mph while Gina is 10 miles due south of the train station and moving north at 50 mph.

(a) What is the distance between Gina and Tommy at the time Tommy sends his text? Remember to include units.

(b) When Tommy sends his text, are he and Gina moving closer together or farther apart? How quickly? You must show your work clearly to earn any credit. Remember to include units.

(c) Let $J(t)$ be the distance between Gina and Tommy t hours after Tommy sends his text. Use the local linearization of $J(t)$ at $t = 0$ to estimate the distance between Gina and Tommy 0.1 hours after Tommy sends his text. Remember to show your work carefully.

Problem 4 (Fall 2016 Final Exam Problem 2)**.** Uri is filling a cone with molten aluminum. The cone is upside-down, so the base is at the top of the cone and the vertex at the bottom, as shown in the diagram. The base is a circular disk with radius 7 cm and the height of the cone is 12 cm. Recall that the volume of a cone is $\frac{1}{3}Ah$, where A is the area of the base and h is the height of the cone (i.e., the vertical distance from the vertex to the base). (Note that the diagram may not be to scale.)

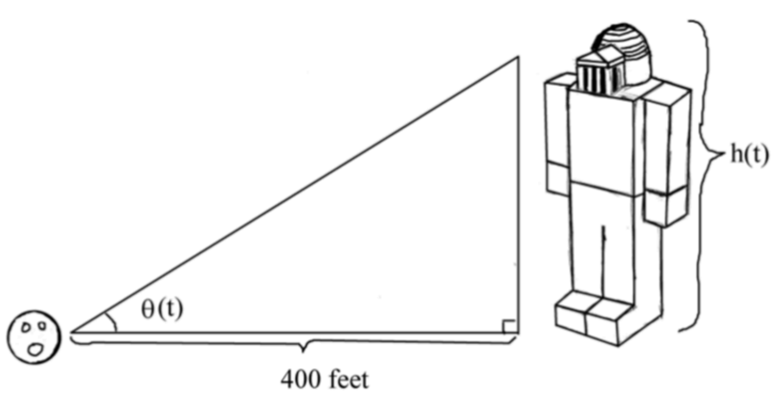


(a) Write a formula in terms of h for the volume V of molten aluminum, in cm^3, in the cone if the molten aluminum in the cone reaches a height of h cm.

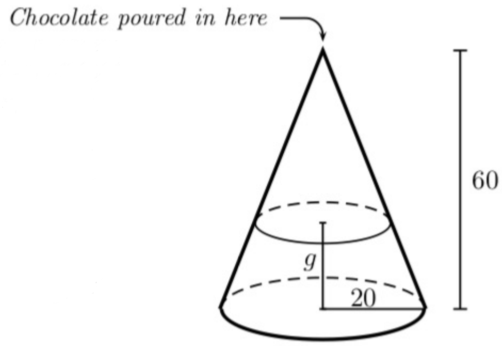
(b) The height of molten aluminum is rising at 3 cm/sec at the moment when the molten aluminum in the cone has reached a height of 11 cm. What is the rate, in cm^3/sec, at which Uri is pouring molten aluminum into the cone at that moment?

(c) The height of molten aluminum is rising at 3 cm/sec at the moment when the molten aluminum in the cone has reached a height of 11 cm. What is the rate, in cm^2/sec, at which the area of the top surface of the molten aluminum is increasing at that moment?

Problem 5. The Rotunda is a building located on The Lawn on the original grounds of the University of Virginia. During the break, you decide to visit. You are sitting on the lawn enjoying a nice sunny day during your break when the rotunda starts moving! As it turns out, Thomas Jefferson designed it to turn into a giant robot to defend the campus. You are watching the head of the robot, the rotunda (which is rising at 10 feet per second) from a safe distance of 400 feet away. How fast is your head angling upwards when the robot is 300 feet tall?



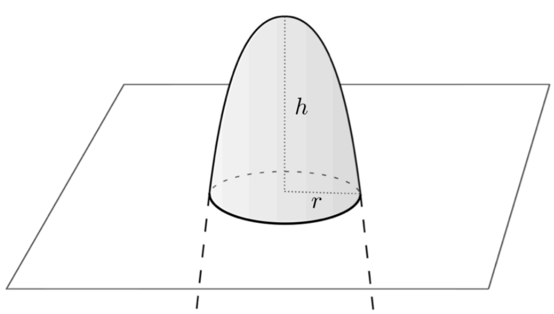
Problem 6 (Winter, 2015 Final Exam Problem 4)**.** Having taken care of Sebastian and sent Erin into the hands of the Illumisqati, King Roderick is pleased that his plan is proceeding well. Our wicked villain decides to relax with a handmade chocolate before he heads to his farmhouse. The process of making the chocolate involves pouring molten chocolate into a mould. The mould is a cone with height 60 mm and base radius 20 mm. Roderick places the mould on the ground and begins pouring the chocolate through the apex of the cone.



(a) Let g be the depth of the chocolate, in mm, as shown in the diagram above. What is the value of g when Roderick has poured a total of 20,000 mm^3 of chocolate into the mould? Show your work carefully, and make sure your answer is accurate to at least two decimal places.

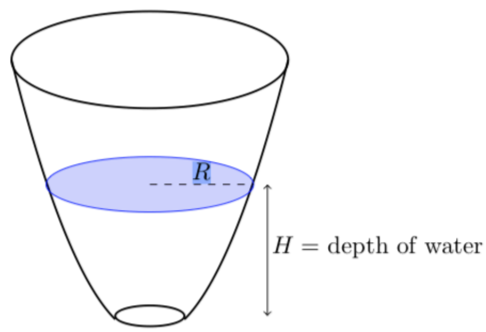
(b) How fast is the depth of the chocolate in the mould (g in the diagram above) changing when Roderick has already poured 20,000 mm3 of chocolate into the mould if he is pouring at a rate of 5,000 mm^3 per second at this time? Show your work carefully and make sure your answer is accurate to at least two decimal places. Be sure to include units.

Problem 7 (Winter 2018 Final Exam Problem 7)**.** A group of meteorologists observe that the sea level is rising by observing a piece of a rock in the sea. Only the tip of the rock is visible, and as the sea water rises, less and less of the rock is above water. Let h and r be the height and radius (in inches), respectively, of the part of the rock that is above the sea. The volume of the rock (in cubic inches) is then given by the formula $V = \frac{\pi}{2}(1 + r^2)h$.



The meteorologists notice that, as the level of the sea is rising, the radius and volume of the rock are changing. A year after they started taking the measurements, the radius and height of the rock are 5 and 46 inches, respectively. They notice that at that time, the radius is decreasing at a rate of 0.05 inches per year, which makes the volume change at a rate of 80 cubic inches per year. At what rate is the height of the rock changing at that time? Be sure to include units. Is the height of the part of the rock that is above the sea increasing or decreasing?

Problem 8 (Fall 2017 Final Exam Problem 6)**.** Water is being poured into a large vase with a circular base. Let $V(t)$ be the volume of water in the vase, in cubic inches, t minutes after the water started being poured into the vase. Let H be the depth of the water in the vase, in inches, and let R be the radius of the surface of the water, in inches. A formula for V in terms of R and H is given by $V = \frac{\pi}{2}H(R^2 + 8)$.



(a) Suppose that the water is being poured into the vase at rate of 300 cubic inches per minute. When the depth of the water is 5 inches, the radius of the surface of the water is 4 inches and the radius is increasing at a rate of 1.2 inches per minute. Find the rate at which the depth of the water in the vase is increasing at that time. Show all your work carefully.

(b) Estimate the instantaneous rate of change of H when $t = 3$ if

t	1.5	2.3	3.0	3.2
H	1.3	1.7	1.9	1.95

Show your work and inlude units.

(c) Recall that R gives the radius of the surface of the water, in inches, t minutes after the water started being poured into the vase. Suppose that R is given by $R = m(t)$ and $m'(3) = 0.7$. Use these facts to complete the following sentence:

After the water has been poured into the vase for three minutes, over the next ten seconds, the radius of the surface of the water