Worksheet 27

Problem 1. The record time for the 100 meter dash is 9.58 seconds, set by Usain Bolt at a race in 2009. Let $v(t)$ be Bolt's velocity, in meters per second, t seconds after Bolt starts the race. Several values of $v(t)$ are shown below.

t	0	0.5	1	1.5	2	2.5	3
$v(t)$	0.6	3.5	5.8	7.7	9.1	10.1	10.6

(a) Assume that Usain keeps his velocity constant in each of the intervals $[0, 0.5)$, $[0.5, 1)$, $[1.5, 2)$, $[2, 2.5)$ and $[2.5, 3)$. Compute the total distance Usain ran in the first 3 seconds.

(b) Now assume instead that Usain keeps his velocity constant in the intervals $(0, 0.5]$, $(0.5, 1]$, $(1.5, 2]$, $(2, 2.5]$ and $(2.5, 3]$ and compute the total distance he ran in this situation.

(c) For each of the situations described in (a) and (b), make a sketch of the graph of the function $v(t)$. Can you represent each of the sums computed in (a) and (b) on the respective graph?

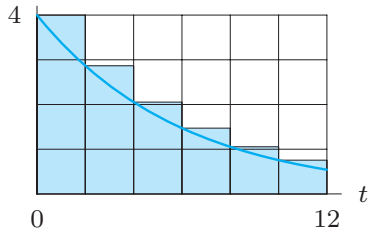
Assume now that $v(t)$ is a continuous increasing function for the first three seconds of the race.

(d) Based on the values provided on the table make a new sketch of $v(t)$.

(e) What information about the graph in (d) do the sums in (a) and (b) provide?

(f) Estimate the error when you compute the area below the graph of $v(t)$ using (a) and (b).

Problem 2. The figure below shows the velocity of a car for $0 \leqslant t \leqslant 12$ and the rectangles used to estimate of the distance traveled.



(a) Do the rectangles represent a left or a right sum?

(b) Do the rectangles lead to an upper or a lower estimate?

(c) What is the value of n?

(d) What is the value of Δt?

(e) Give an approximate value for the estimate.

Problem 3. The velocity $v(t)$ in the table below is decreasing for $2 \leqslant t \leqslant 12$.

t	2	4	6	8	10	12
$v(t)$	44	42	41	40	37	35

Using $n = 5$ subdivisions to approximate the total distance traveled, find

(a) An upper estimate.

(b) A lower estimate.

Distance as area

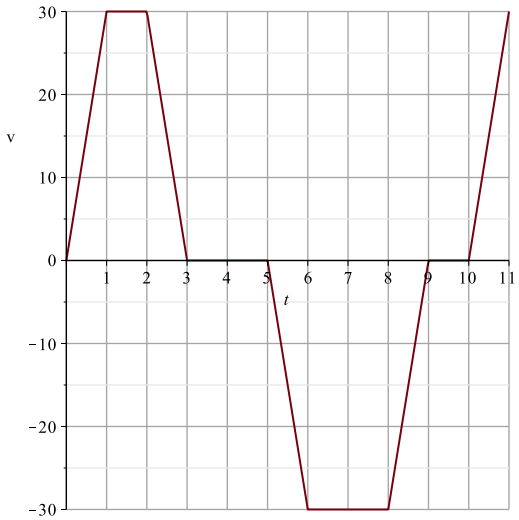
Problem 4. A particle's velocity is given by $v(t)$, where

$$\text{(a)}\quad v(t) = \begin{cases} 2, & 0 \leqslant t \leqslant 3 \\ -8 + 2t, & 3 < t \leqslant 6 \end{cases} \qquad \text{(b)}\quad v(t) = 3t^2, \quad 0 \leqslant t \leqslant 6.$$

(i) For (a), compute the particle's displacement (signed area under the curve) between $t = 0$ and $t = 6$ using geometry.

(ii) For (b), estimate the displacement of the particle, using left and right sums with $n = 6$.

Problem 5. A bicyclist starts from home and rides back and forth along a straight east/west highway. Her velocity $y = v(t)$ at time t (in minutes) measured in feet per second is given below (positive velocities indicate travel toward the east, negative toward the west).



(a) On what time intervals is she stopped?

(b) How far from home is she the first time she stops, and in what direction?

(c) At what time does she bike past her house?

(d) If she maintains her velocity at $t = 11$, how long will it take her to get back home?

Problem 6. The table below gives the expected growth rate, $g(t)$, in ounces per week, of the weight of a baby in its first 54 weeks of life. Assume for this problem that $g(t)$ is a decreasing function.

week t	0	9	18	27	36	45	54
growth rate $g(t)$	6	6	4.5	3	3	3	2

(a) Using six subdivisions, find an overestimate and an underestimate for the total weight gained by a baby over its first 54 weeks of life.

(b) How frequently over the 54 week period would you need the data for $g(t)$ to be measured to find overestimates and underestimates for the total weight gain over this time period that differ by 8 oz?