Worksheet 28

Problem 1. Consider the integral

$$\int_0^1 x\,dx.$$

(a) Compute (in exact form) the left-hand sum of this integral using n subdivisions.

(b) Compute (in exact form) the right-hand sum of this integral using n subdivisions.

(c) Evaluate the limits of the quantities you computed in (a) and (b) when $\Delta t \to 0$.

Problem 2. Find the integral

$$\int_0^{10} x - 5\ dx$$

by finding the area of the region between the curve and the horizontal axis.

Problem 3. Consider the function

$$f(x) = \begin{cases} 1 - x, & 0 \leqslant x \leqslant 1, \\ x - 1, & 1 < x \leqslant 2. \end{cases}$$

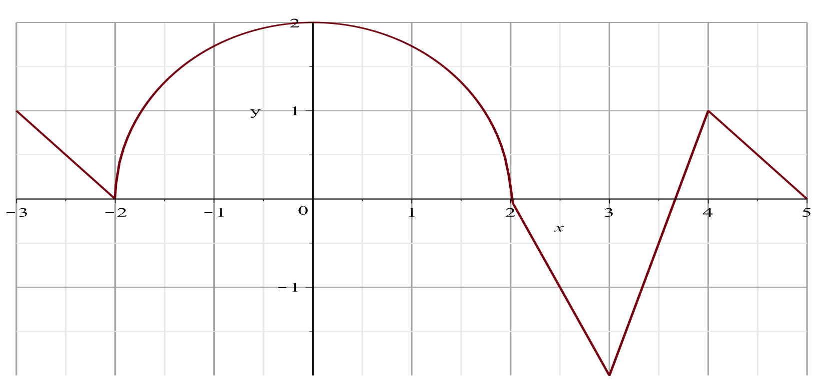
(a) Sketch the graph of f.

(b) Find $\displaystyle\int_0^2 f(x)\,dx$.

(c) Find the 4-term left Riemann sum approximation of the definite integral you just computed. How does your approximation compare to the exact value?

Signed area

Problem 4. The plot below shows $y = g(x)$.

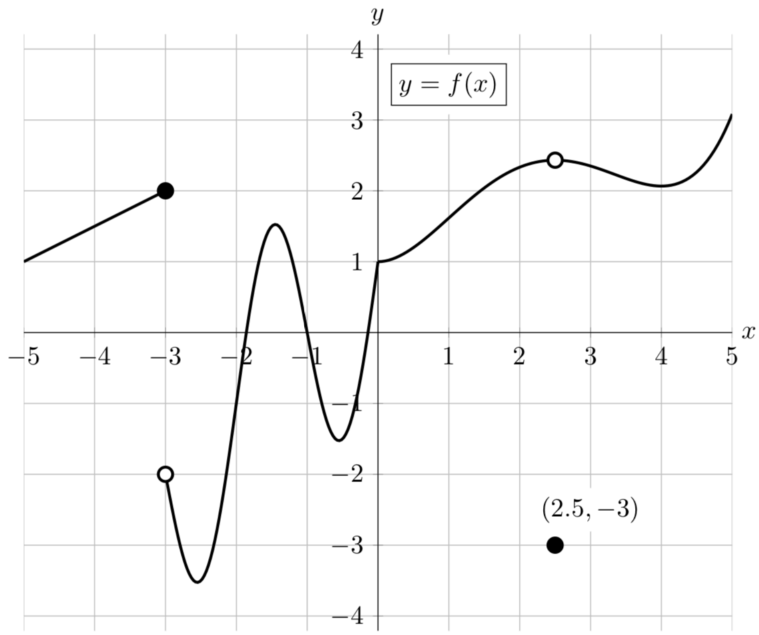


Find the exact value of

(a) The definite integral $\displaystyle\int_{-3}^{5} g(x)\,dx$.

(b) The definite integral $\displaystyle\int_{-3}^{5} |g(x)|\,dx$.

Problem 5 (Winter 2016 Final Exam Problem 1)**.** A portion of the graph of a function f is shown below.



(a) For which of the values of c is $\lim_{x\to c^-} f(x) = f(c)$?

$c = -3$ $\qquad$ $c = -1$ $\qquad$ $c = 0$ $\qquad$ $c = 1$ $\qquad$ $c = 2.5$ $\qquad$ none

(b) For which of the following values of c is $f(x)$ continuous at $x = c$?

$c = -3$ $\qquad$ $c = -1$ $\qquad$ $c = 0$ $\qquad$ $c = 1$ $\qquad$ $c = 2.5$ $\qquad$ none

(c) For which of the following values of c does f appear to be differentiable at $x = c$?

$c = -3$ $\qquad$ $c = -1$ $\qquad$ $c = 0$ $\qquad$ $c = 1$ $\qquad$ $c = 2.5$ $\qquad$ none

(d) Rank the following quantities in order from least to greatest:

I. The number 0.

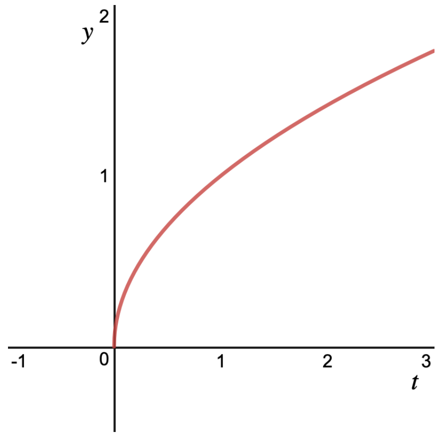
II. $f(1)$.

III. $\int_{-1}^{1} f(x)dx$.

IV. The left-hand Riemann sum with 2 subintervals for $\int_{-1}^{1} f(x)dx$.

V. The right-hand Riemann sum with 2 subintervals for $\int_{-1}^{1} f(x)dx$.

Problem 6. We want to compute $\int_0^2 \sqrt{t}\, dt$. Below is a portion of the graph of $f(t) = \sqrt{t}$.



(a) For this integral, are left sums always overestimates, always underestimates, or could they be either? What about right sums?

(b) Use a Riemann sum with 5 equal subdivisions to find a lower estimate for the integral. Show your answer to three decimal places.

(c) Use a Riemann sum with 5 equal subdivisions to find an upper estimate for the integral. Show your answer to three decimal places.

(d) Repeat (b) and (c) with 10 equal subdivisions. Show your answers to three decimal places.

Problem 7. For each of the following statements, must the statement be true for all continuous functions $f(x)$ and $g(x)$? Explain your answer.

(a) $\int_0^2 f(x)\, dx \leqslant \int_0^3 f(x)\, dx$.

(b) $\int_0^2 f(x)\, dx = \int_0^2 f(t)\, dt$.

(c) If $\int_2^6 f(x)\, dx \leqslant \int_2^6 g(x)\, dx$, then $f(x) \leqslant g(x)$ for all $2 \leqslant x \leqslant 6$.