Worksheet 30

Warm-up question

Average value of the function f between a and b =

Problem 1 (Fall 2016 Final Exam Problem 5)**.** The table below gives several values of a function $q(u)$ and its first and second derivatives. Assume that all of $q(u)$, $q'(u)$, and $q(u)$ are defined and continuous for all real numbers u.

u	0	1	2	3	4	5	6
$q(u)$	30	23	19	20	24	25	24
$q'(u)$	0	-6	-2	1	3	1	-2
$q''(u)$	-9	5	4	3	2	-5	0

Compute each of the following. Do not give approximations. If it is not possible to find the value exactly, write not possible.

(a) Compute $\displaystyle\int_5^2 q''(t)\,dt$.

(b) Suppose $q(u)$ is an even function. Compute $\displaystyle\int_{-5}^5 (q'(u)+7)\,dt$.

(c) Suppose $q(u)$ is an even function. Compute $\displaystyle\int_{-5}^5 q(u)\,dt$.

Problem 2. Suppose n is a positive integer, f is a decreasing, continuous function on $[2,6]$, the value of the left Riemann sum with n equal subdivisions for $\displaystyle\int_2^6 f(x)\,dx$ is A, and $f(2) = f(6)+8$. Circle all the statements that must be true.

(a) A is an overestimate for $\displaystyle\int_2^6 f(x)\,dx$.

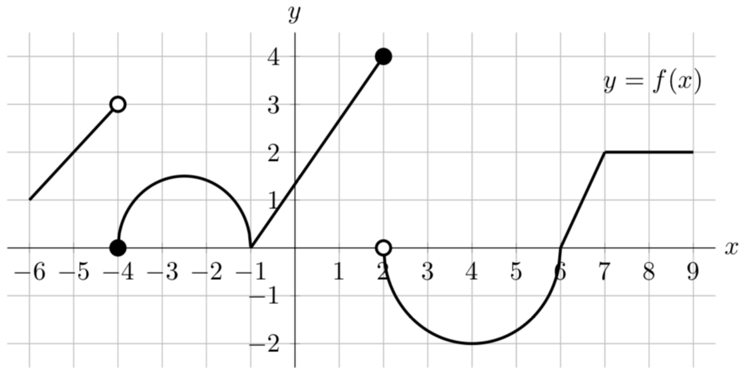
(b) $\displaystyle\int_2^6 f(x)\,dx = 8$.

(c) $\displaystyle\int_1^5 f(x+1)\,dx = \int_2^6 f(x+1)\,dx$.

(d) The left Riemann sum for $\displaystyle\int_2^6 (f(x))^2\,dx$ with n equal subdivisions is A^2.

(e) none of these

Problem 3 (Winter 2017 Final Exam Problem 2)**.** The graph of f shown below consists of lines and semicircles.



Use the graph above to calculate the answers to the following questions. Give your answers as exact values. You do not need to show work. If any of the answers cannot be found with the information given, write *nei*.

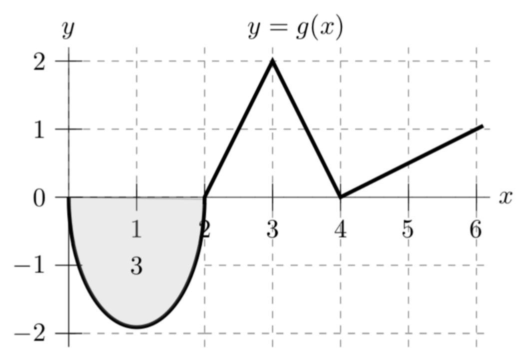
(a) Find the average value of $f(x)$ on $[-4, 2]$.

(b) Find the value of $\displaystyle\int_4^9 |f(z)|\, dz$.

(c) Find the value of $4 \leqslant T \leqslant 9$ such that $\displaystyle\int_4^T |f(z)|\, dz = 0$.

(d) Find the value of $\displaystyle\int_{-8}^{-7} f(x+2) + 1\, dx$.

Problem 4 (Winter 2016 Final Exam Problem 6)**.** A portion of the graph of a continuous function $g(x)$ is shown below. Assume that the area of the shaded region is 3 (as indicated on the graph), and note that $g(x)$ is piecewise linear for $2 < x < 6$.



(a) Find $\displaystyle\int_0^6 g(x)\, dx$.

(b) Find $\displaystyle\int_0^2 5 - 4g(x)\, dx$.

(c) Suppose $C(x) = \ln(g(x))$. Find $C'(2.5)$.

(d) Find $\displaystyle\int_2^4 g(x+2) - g(x-2)\, dx$.

Problem 5 (Winter 2014 Final Exam Problem 4)**.** One of the ways Captain Christina likes to relax in her retirement is to go for long walks around her neighborhood. She has noticed that early every Tuesday morning, a truck delivers butter to a local bakery famous for its cookie dough. Consider the following functions:

- Let $C(b)$ be the bakery's cost, in dollars, to buy b pounds of butter.
- Let $K(b)$ be the amount of cookie dough, in cups, the bakery makes from b pounds of butter.
- Let $u(t)$ be the instantaneous rate, in pounds per hour, at which butter is being unloaded t hours after 4 am.

(a) Interpret $K(C^{-1}(10)) = 20$ in the context of this problem.

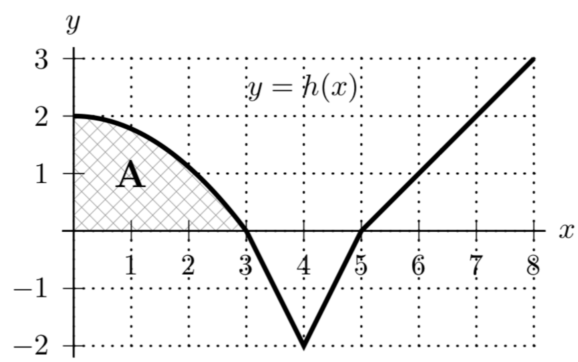
(b) Interpret $\int_5^{12} K'(b)\,db = 40$ in the context of this problem.

(c) Give a single mathematical equality involving the derivative of C which supports the following claim: It costs the bakery approximately \$0.70 less to buy 14.8 pounds of butter than to buy 15 pounds of butter.

(d) Assume that $u(t) > 0$ and $u'(t) < 0$ for $0 \leqslant t \leqslant 4$ and that $u(2) = 800$. Rank the following quantities in order from least to greatest.

I. 0 II. 800 III. $\int_1^2 u(t)\,dt$ IV. $\int_2^3 u(t)\,dt$

Problem 6 (Winter 2014 Final Exam Problem 1)**.** The graph of a function $h(x)$ is shown below. The area of the shaded region A is 4, and $h(x)$ is piecewise linear for $3 \leqslant x \leqslant 6$.



Compute each of the following. If there is not enough information to compute a value exactly, write *not enough info*.

(a) Find $\int_0^3 h(t) + 2\,dt$.

(b) Find the average value of $h(x)$ on the interval $[0, 4]$.

(c) Let $J(x) = \sin(\pi h(x))$. Find $J'(3.5)$.

(d) Let $g(x) = e^x$. Find $\int_6^7 g'(x)h(x) + g(x)h'(x)\,dx$.