Worksheet 31

Warm-up question

The function $F(x)$ is an antiderivative of $f(x)$ if

If $F(x)$ is an antiderivative of the differentiable function $f(x)$,

(a) If f is positive on an interval, then F is ____________________ on that interval.

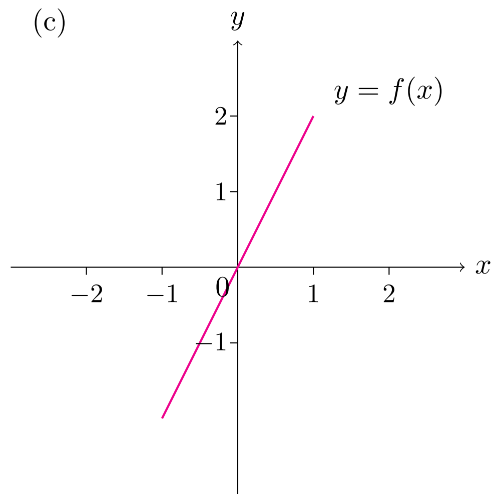
(b) If f is increasing on an interval, then F is ____________________ on that interval.

Problem 0. For each of the given functions f, sketch the graphs of its antiderivatives F, G, H and K such that $F(0) = 0$, $G(0) = 1$, $H(0) = 2$ and $K(0) = -2$.

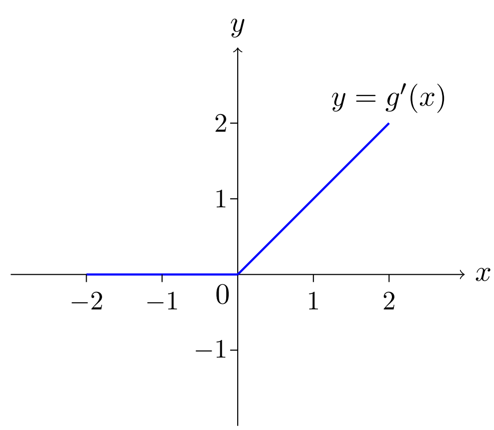
(a) f is the constant function $f(x) = 0$.

(b) f is the constant function $f(x) = 1$.

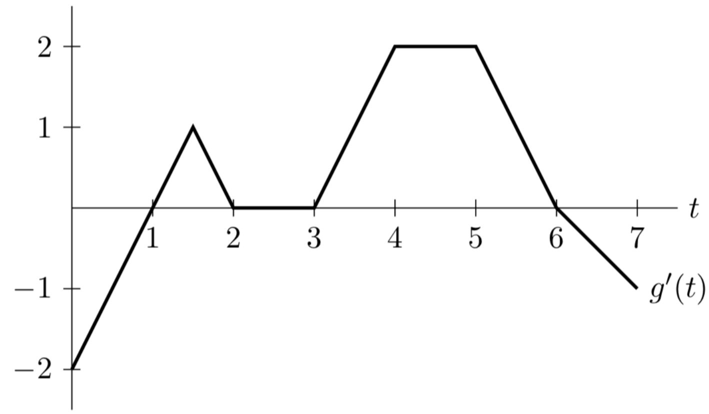
(c)



Problem 1. The function g is defined on the interval $[-2, 2]$ and has $g(-1) = 1$. Below is the graph of its derivative $g'(x)$. Sketch the graph of g.



Problem 2 (Fall 2013 Final Exam Problem 3). The function $g(t)$ is the volume of water in the town water tank, in thousands of gallons, t hours after 8 A.M. A graph of $g'(t)$ is shown below. Note that $g'(t)$ is a piecewise-linear function. Suppose that $g(3) = 1$.

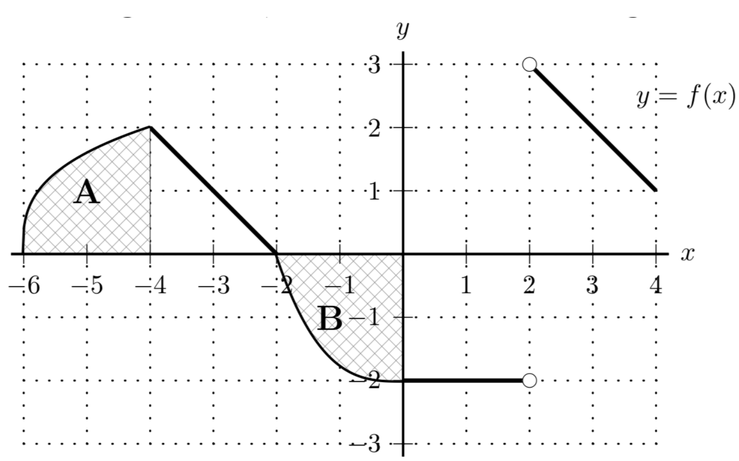


(a) Write an integral which represents the average rate of change, in thousands of gallons per hour, of the volume of water in the tank between 9 A.M. and 1 P.M. Compute its exact value.

(b) At what time does the tank have the most/least water in it?

(c) Sketch a graph of $g(t)$ and give both coordinates of the point on the graph at $t = 7$.

Problem 3 (Fall 2014 Final Exam Problem 4). A portion of the graph of $y = f(x)$ is shown below. The area of the region A is 3, and the area of the region B is 3. Let $F(x)$ be the continuous antiderivative of $f(x)$ with $F(0) = 1$ whose domain includes the interval $-6 \leqslant x \leqslant 4$.

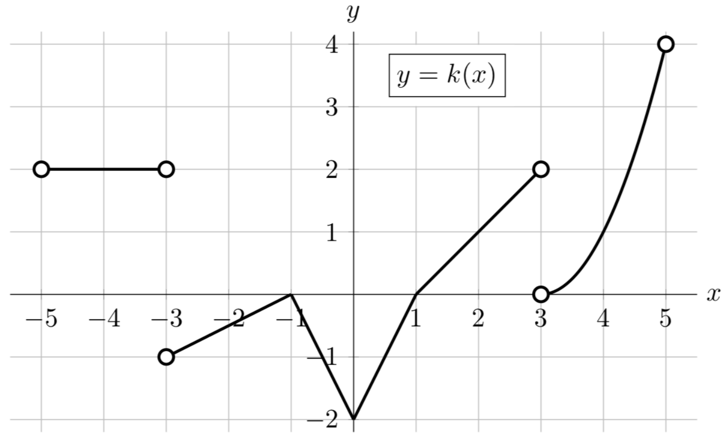


(a) For what value(s) of x with $-6 < x < 4$ does $F(x)$ have local extrema?

(b) Sketch the graph of $y = F(x)$ on the interval $-6 \leqslant x \leqslant 4$. Pay close attention to the following:

- the value of $F(x)$ at each of $x = -6, -4, -2, 0, 2, 4$;
- where F is/is not differentiable;
- where F is increasing/decreasing/constant;
- the concavity of the graph of $y = F(x)$.

Problem 4 (Fall 2016 Final Exam Problem 11)**.** A portion of the graph of k is shown below. Note that for $3 < x < 5$, the graph of $k(x)$ is a portion of the graph obtained by shifting $y = x^2$ three units to the right. Let $K(x)$ be a continuous antiderivative of k passing through the point $(-1, 1)$.



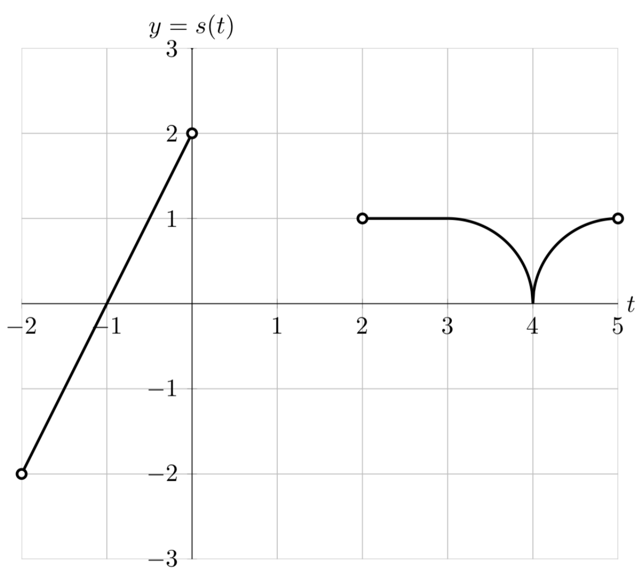
(a) Use the graph to complete the table with exact values of K.

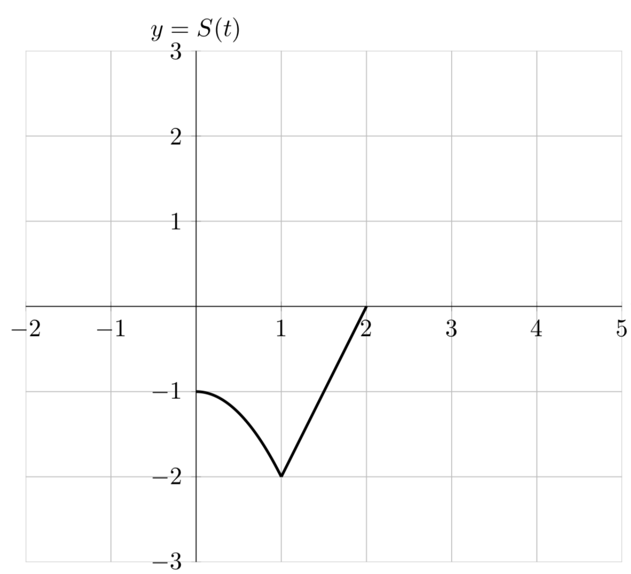
x	-5	-3	-1	1	3	5
$K(x)$						

(b) Sketch a detailed graph of $y = K(x)$ for $-5 < x < 5$. Pay close attention to the following:

- where $K(x)$ is and is not differentiable,
- the values of $K(x)$ you found in the table above,
- where $K(x)$ is increasing/decreasing/constant, and the concavity of $K(x)$.

Problem 5 (Fall 2017 Final Exam Problem)**.** A portion of the graphs of two functions $y = s(t)$ and $y = S(t)$ are shown below. Suppose that $S(t)$ is the continuous antiderivative of $s(t)$ passing through the point $(0, -1)$. Note that the graphs are linear anywhere they appear to be linear, and that on the intervals $(3, 4)$ and $(4, 5)$, the graph of $s(t)$ is a quarter circle.





(a) Use the portions of the graphs to fill in the exact values of $S(t)$ in the table below.

t	-2	-1	0	2	3	5
$S(t)$						

(b) Sketch the missing portions of both s and S over the interval $-2 < t < 5$. Pay attention to:

- the values of $S(t)$ from the table above;
- where S is and is not differentiable;
- the concavity of the graph $y = S(t)$.
- where S and s are increasing/decreasing/constant;

Problem 6 (Winter 2016 Final Exam Problem 10)**.** Which of the following is an antiderivative of the function $f(x) = \cos(x)$? Circle all the correct options.

(a) $\frac{\cos(x)}{2}$

(b) $\sin(x) + 5$

(c) $\cos\left(x - \frac{\pi}{2}\right)$

(d) $\ln\left(3e^{\sin(x)}\right)$

(e) $19 - \sin(x)$

(f) None of these

Problem 7 (Fall 2017 Final Exam Problem 9)**.** Which of the following is an antiderivative of the function $f(x) = \frac{1}{x} + \cos(x)$? Circle all the correct options.

(a) $-\frac{1}{x^2} - \sin(x)$

(b) $\ln(5x) + \sin(x)$

(c) $\ln(x) + \sin(x) - 20$

(d) $\ln\left(\frac{1}{x}\cos(x)\right)$

(e) $\frac{1}{x^2} + \sin(x)$

(f) None of these

Problem 8 (Winter 2013 Final Exam Problem 9)**.** The number p is a constant. Which of the following is an antiderivative of $g(x) = \ln(x + p)$?

(a) $G(x) = \frac{p}{x+p}$.

(b) $G(x) = \frac{1}{x+p}$.

(c) $G(x) = (x + p)\ln(x + p) - x$.

(d) $G(x) = \frac{\ln(x+p)}{p} - x$.

(e) $G(x) = x^2 \ln(x + p) - x$.

Problem 9 (Winter 2015 Final Exam Problem 11)**.** Suppose that w and r are continuous functions on $(-\infty, \infty)$, $W(x)$ is an invertible antiderivative of $w(x)$, and $R(x)$ is an antiderivative of $r(x)$. Which of the following statements must be true?

(a) $W(x) + R(x) + 2$ is an antiderivative of $w(x) + r(x)$.

(b) $W(x) + R(x) + 2$ is an antiderivative of $w(x) + r(x) + 2$.

(c) $\cos(W(x))$ is an antiderivative of $\sin(w(x))$.

(d) $e^{W(x)}$ is an antiderivative of $w(x)e^{w(x)}$.

(e) $e^{R(x)}$ is an antiderivative of $r(x)e^{R(x)}$

(f) If w is never zero, then $W^{-1}(R(x))$ is an antiderivative of $\frac{r(x)}{w(W^{-1}(R(x)))}$.

(g) None of these