

Worksheet 32

Warm-up questions

$$\begin{array}{ll} \text{If } k \text{ is a constant, } \int k \, dx = & \int \sin(x) \, dx = \\ \text{If } n \neq -1, \int x^n \, dx = & \int \cos(x) \, dx = \\ \int \frac{1}{x} \, dx = & \int e^x \, dx = \end{array}$$

Problem 0. Find the following indefinite integrals.

$$\begin{array}{llllll} \text{(a)} \int \frac{8}{\sqrt{u}} \, du & \text{(c)} \int 2 + \cos t \, dt & \text{(e)} \int (x+3)^2 \, dx & \text{(g)} \int \frac{x+1}{x} \, dx & \text{(i)} \int 4t + \frac{1}{t} \, dt \\ \text{(b)} \int e^x + x^e \, dx & \text{(d)} \int 7e^x \, dx & \text{(f)} \int t^3(t^2+1) \, dx & \text{(h)} \int \sqrt{x^3} - \frac{2}{x} \, dx & \text{(j)} \int \sin(3x) \, dx \end{array}$$

Problem 1. Evaluate the definite integrals exactly using the Fundamental Theorem of Calculus.

$$\begin{array}{ll} \text{(a)} \int_0^3 (x^2 + 4x + 3) \, dx & \text{(c)} \int_0^2 3e^t \, dt \\ \text{(b)} \int_0^{\frac{\pi}{4}} \sin \theta \, d\theta & \text{(d)} \int_1^2 \frac{1+y^2}{y} \, dy \end{array}$$

Problem 2 (Winter 2016 Final Exam Problem 10). Which of the following is an antiderivative of the function $f(x) = \cos(x)$? Circle all the correct options.

$$\begin{array}{lll} \text{(a)} \frac{\cos(x)}{2} & \text{(c)} \cos\left(x - \frac{\pi}{2}\right) & \text{(e)} 19 - \sin(x) \\ \text{(b)} \sin(x) + 5 & \text{(d)} \ln(3e^{\sin(x)}) & \text{(f)} \text{None of these} \end{array}$$

Problem 3 (Fall 2017 Final Exam Problem 9). Which of the following is an antiderivative of the function $f(x) = \frac{1}{x} + \cos(x)$? Circle all the correct options.

$$\begin{array}{lll} \text{(a)} -\frac{1}{x^2} - \sin(x) & \text{(c)} \ln(x) + \sin(x) - 20 & \text{(e)} \frac{1}{x^2} + \sin(x) \\ \text{(b)} \ln(5x) + \sin(x) & \text{(d)} \ln\left(\frac{1}{x} \cos(x)\right) & \text{(f)} \text{None of these} \end{array}$$

Problem 4 (Winter 2013 Final Exam Problem 9). The number p is a constant. Which of the following is an antiderivative of $g(x) = \ln(x+p)$?

$$\begin{array}{ll} \text{(a)} G(x) = \frac{p}{x+p}. & \text{(d)} G(x) = \frac{\ln(x+p)}{p} - x. \\ \text{(b)} G(x) = \frac{1}{x+p}. & \\ \text{(c)} G(x) = (x+p) \ln(x+p) - x. & \text{(e)} G(x) = x^2 \ln(x+p) - x. \end{array}$$

Problem 5 (Winter 2015 Final Exam Problem 11). Suppose that w and r are continuous functions on $(-\infty, \infty)$, $W(x)$ is an invertible antiderivative of $w(x)$, and $R(x)$ is an antiderivative of $r(x)$. Which of the following statements must be true?

- (a) $W(x) + R(x) + 2$ is an antiderivative of $w(x) + r(x)$.
- (b) $W(x) + R(x) + 2$ is an antiderivative of $w(x) + r(x) + 2$.
- (c) $\cos(W(x))$ is an antiderivative of $\sin(w(x))$.
- (d) $e^{W(x)}$ is an antiderivative of $w(x)e^{w(x)}$.
- (e) $e^{R(x)}$ is an antiderivative of $r(x)e^{R(x)}$.
- (f) If w is never zero, then $W^{-1}(R(x))$ is an antiderivative of $\frac{r(x)}{w(W^{-1}(R(x)))}$.
- (g) None of these