Worksheet 5

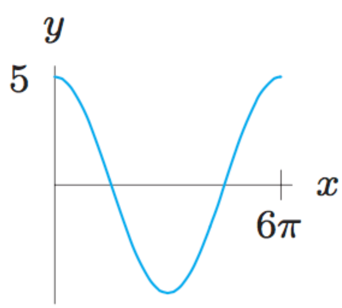
Important remark: In this worksheet, all the angles under consideration are in *radians*.

Warm-up questions

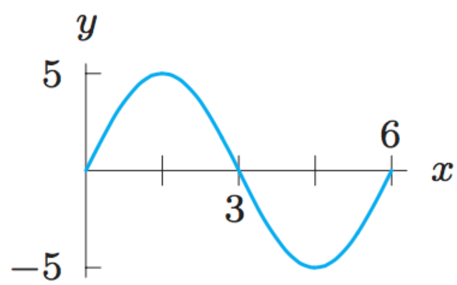
The sinusoidal function $f(t) = C + A\sin(Bt)$ has amplitude __________ and period __________.

The sinusoidal function $g(t) = C + A\cos(Bt)$ has amplitude __________ and period __________.

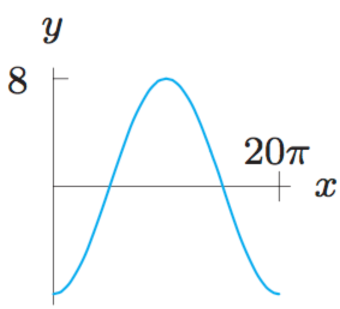
Problem 1. Find a possible formula for each of the graphs below.



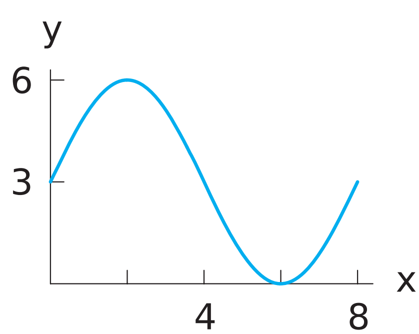
(a)



(b)



(c)



(d)

Problem 2 (Fall 2012 Exam 1)**.** The population of squirrels in Ann Arbor oscillates sinusoidally between a low of 4.1 thousand on January 1 and a high of 5.4 thousand on July 1. Let $P(t)$ be the population, in thousands, of squirrels in Ann Arbor t months since January 1.

(a) Draw the graph of the function $P(t)$ on the interval $[0, 14]$. Remember to label your axes and make sure important features of the graph are clear.

(b) Use your graph to find a formula for $P(t)$.

(c) What are the period and amplitude of $P(t)$?

Inverse trigonometric functions

Problem 3. This problem introduces the arccosine function, or inverse cosine, denoted by $\cos^{-1}$ on most calculators.

(a) Using a calculator set in radians, complete the table of values, to two decimal places, of the function $g(x) = \arccos x$.

x	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
$\arccos x$											

(b) Sketch the graph of $g(x) = \arccos x$.

(c) Why is the domain of the arccosine the same as the domain of the arcsine?

(d) What is the range of the arccosine?

(e) Why is the range of the arccosine *not* the same as the range of the arcsine?

Problem 4. Find a solution to the equation if possible. Give the answer in exact form and in decimal form.

(a) $2 = 5\sin(3x)$

(b) $1 = 8\cos(2x+1) - 3$

(c) $8 = 4\tan(5x)$

(d) $1 = 8\tan(2x+1) - 3$

(e) $8 = 4\sin(5x)$

Problem 5. The desert temperature, H, oscillates daily between 40°F at 5 am and 80°F at 5pm.

(a) Write a possible formula for H in terms of t, measured in hours from 5 am.

(b) Determine the number of hours in each day (both exact and approximate) that the temperature is above 55°F.

Problem 6 (Winter 2012 Exam 1)**.** Enjoying breakfast outdoors in a coastal Mediterranean town, Tommy notices a ship that is anchored offshore. The ship is stationed above a reef which lies below the surface of the water, and a series of waves causes its height to oscillate sinusoidally with a period of 6 seconds. When Tommy begins observing, the hull of the ship is at its highest point, 20 feet above the reef. After 1.5 seconds, the hull is 11 feet above the reef.

(a) Write a function $h(t)$ that gives the height of the ship's hull above the reef t seconds after Tommy begins observing.

(b) According to your function, will the hull of the ship hit the reef? Explain.

Problem 7 (Fall 2013 Exam 1)**.** After the success of his new bacon-flavored soda, Louis wants to try making a flavor that customers will find more refreshing in the hot summer months. Louis notices daily sales of his new spearmint soda vary seasonally. Sales reach a high of \$300 around August 1 and a low of \$120 around February 1st. Suppose that daily sales of the soda (in dollars) can be modeled by a sinusoidal function $S(t)$ where t is the time in months since January 1. Note that August 1st is seven months after January 1st.

(a) What are the period and amplitude of the function $S(t)$?

(b) Write a formula for the function $S(t)$.

Problem 8 (Winter 2018 Exam 1). A company designs chambers whose interior temperature can be controlled. Their chambers come in two models: Model A and Model B.

(a) The temperature in Model A goes from its minimum temperature of -3°C to its maximum temperature of 15°C and returning to its minimum temperature three times each day. The temperature of this chamber at 10 am is 15°C. Let A(t) be the temperature (in °C) inside this chamber t hours after midnight. Find a formula for A(t) assuming it is a sinusoidal function.

(b) Let B(t) be the temperature (in °C) inside Model B t hours after midnight,

$$B(t) = 5 - 3\cos\left(\frac{3}{7}t + 1\right).$$

Find the two smallest positive values of t at which the temperature in the chamber is 6°C. Your answer must be found algebraically. *Show all your work and give your answers in exact form.*