Worksheet 6

Warm-up questions

Every exponential _____ function eventually dominates every _____.

Consider the rational function $r(x) = \frac{p(x)}{q(x)}$. If the polynomials $p(x)$ and $q(x)$ have no common zeroes, then:

- Zeros of $p(x)$ give rise to _____.
- Zeros of $q(x)$ give rise to _____.

When does $r(x)$ have an horizontal asymptote?

Polynomials, exponentials and rational functions

Problem 1. Which function dominates as $x \to \infty$?

(a) e^x or $10^{1000}\ln(x)$ (b) $1000x^4$ or $0.2x^5$ (c) $10e^{0.1x}$ or $5000x^2$

(d) $100x^5$ or 1.05^x (e) x^4 or $\ln(x)$ (f) e^x or 2.71^x

Problem 2. Which of the following functions have the given properties?

(a) $y = \frac{x^2-2}{x^2+2}$ (b) $y = \frac{x^2+2}{x^2-2}$ (c) $y = (x-1)(1-x)(x+1)^2$

(d) $y = x^3 - x$ (e) $y = x - \frac{1}{x}$ (f) $y = (x^2-2)(x^2+2)$

(i) A polynomial of degree 3.

(ii) A polynomial of degree 4.

(iii) A rational function that is not a polynomial.

(iv) Exactly two distinct zeros.

(v) Exactly one vertical asymptote.

(vi) More than two distinct zeros.

(vii) Exactly two vertical asymptotes.

(viii) A horizontal asymptote.

Problem 3. Exercises 41–44 in Section 1.6 the textbook.

Problem 4 (Fall 2017 Exam 1)**.** Consider the rational function r defined by

$$r(x) = \frac{3(x-\sqrt{2})(\pi x+7)^2(x+1)}{(x+1)(x-\sqrt{3})}.$$

For all of the following parts of this problem, leave your answers in exact form.

(a) What is the domain of $r(x)$?

(b) Find the equations of all vertical asymptotes of $r(x)$. If there are none, write none.

(c) Let $p(x) = x^2 + 1.2x - 5$. Find the equations of all horizontal asymptotes of $\frac{r(x)}{p(x)}$. If there are none, write NONE. Show your work or reasoning to justify your answer.

Problem 5 (Winter 2017 Exam 1)**.** A group of students planted a pine tree and an oak tree alongside the Diag. Let $P(t)$ and $O(t)$ be the height (in feet) of the pine and the oak t years after they were planted, where

$$P(t) = 170 - 165A^{-0.02t} \text{ and } O(t) = \frac{140}{2 + 100e^{-0.3t}}$$

where $A > 1$ is a constant. For this problem, your answers should be in exact form or accurate up to the first two decimal places.

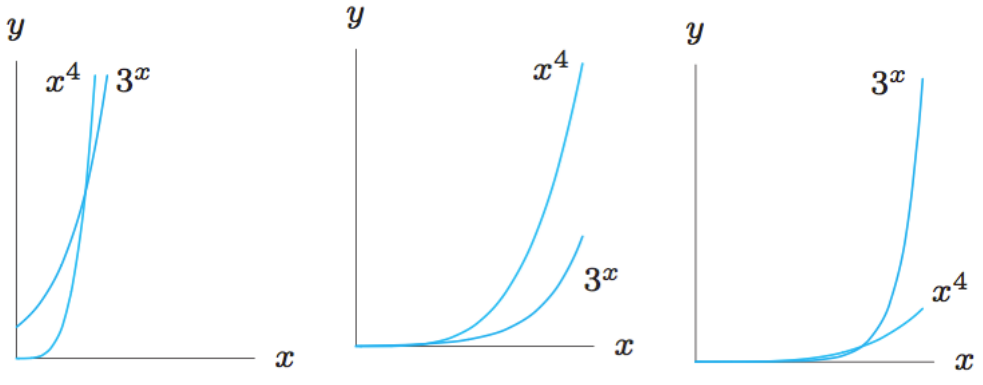
(a) How tall (in feet) were each of the trees when they were planted?

(b) Ten years after the trees were planted, the height of the pine was 38 ft. Find the value of A. Find your answer algebraically and show all your work.

(c) How many years after being planted does it take the oak to be 38 ft? Find your answer algebraically and show all your work.

Finding the right magnifying glass.

Problem 6. Use a graphing calculator to graph $y = x^4$ and $y = 3^x$. Determine approximate domains and ranges that give each of the graphs in the figure below.



(a) (b) (c)