Worksheet 7

Problem 1 (Fall 2017 Exam 1)**.** If $q(x) = \frac{2e^{kx}}{1+2^x}$, find all values of k such that $\lim\limits_{x\to\infty} q(x) = 0$. If there are none, write NONE. Show your work or reasoning to justify your answer.

Problem 2 (Winter 2018 Exam 1)**.**

(a) Let $p(x)$ be a polynomial satisfying all the following properties:

(i) $p(x) = 0$ only at $x = -2, 0, 3$.

(ii) $\lim\limits_{x\to-\infty} p(x) = -\infty$ and $\lim\limits_{x\to\infty} p(x) = -\infty$.

Find one possible formula for $p(x)$. There may be more than one correct answer.

(b) Let $h(x)$ be a rational function satisfying all the following properties:

(i) $\lim\limits_{x\to 2} h(x) = 0$ and h is not defined at $x = 2$.

(ii) $\lim\limits_{x\to\infty} h(x) = 0$.

Problem 3 (Winter 2017 Exam 1)**.** Find all real numbers B and positive integers k such that the rational function

$$H(x) = \frac{9 + x^k}{16 - Bx^3}$$

satisfies the following two conditions:

- $H(x)$ has a vertical asymptote at $x = 2$.
- $\lim\limits_{x\to\infty} H(x)$ exists.

Problem 4 (Winter 2016 Exam 1)**.** Consider the function f(x) defined by

$$f(x) = \begin{cases} xe^{Ax} + B & \text{if } x < 3 \\ C(x-3)^2 & \text{if } 3 \leqslant x \leqslant 5 \\ \frac{130}{x} & \text{if } x > 5 \end{cases}$$

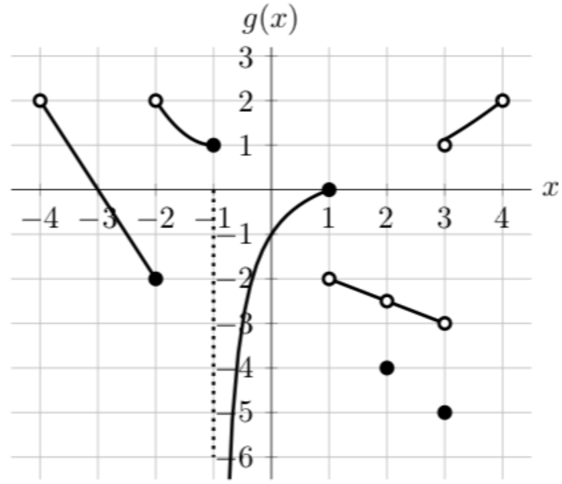
Suppose $f(x)$ satisfies all of the following:

- $f(x)$ is continuous at $x = 3$.
- $\lim\limits_{x\to 5^+} f(x) = 2 + \lim\limits_{x\to 5^-} f(x)$.
- $\lim\limits_{x\to-\infty} f(x) = -4$.

Find the values of A, B and C. You must give exact answers.

Problem 5 (Winter 2018 Exam 1)**.** Consider the functions $f(x)$ and $g(x)$ given by the formula and graph below.

$$f(x) = \begin{cases} 3x^2 - 2x^2 & \text{for } x \leqslant 1 \\ x^3 + 1 & \text{for } x > 1 \end{cases}$$



(a) Circle the correct answer(s) in each of the following questions.

(i) At which of the following values of x is the function $g(x)$ not continuous?

$x = -3$ $\quad$ $x = -1$ $\quad$ $x = 0$ $\quad$ $x = 2$ $\quad$ $x = 3.5$

(ii) At which of the following values of x is the function $f(x) + g(x)$ continuous?

$x = -2$ $\quad$ $x = -1$ $\quad$ $x = 0$ $\quad$ $x = 1$ $\quad$ $x = 2$

Note that $g(x)$ is linear on the intervals $(-4, -2)$, $(1, 2)$ and $(2, 3)$. All your answers below should be exact. If any of the quantities do not exist, write DNE.

(b) Find $\lim_{x \to 2} (2f(x) + g(x))$.

(c) Find $\lim_{x \to \infty} \dfrac{f(2x)}{x^3}$.

(d) Find $\lim_{x \to \infty} g(x^2 e^{-x} + 3)$.

(e) For which values of p does $\lim_{x \to p^+} g(x) = 1$?

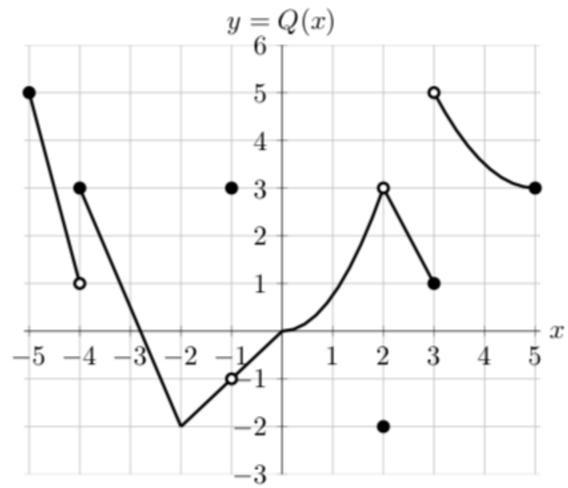
(f) Find $\lim_{x \to -1^-} f(-x)$.

Problem 6. Consider the function

$$N(u) = \begin{cases} e + 3^{u^2+k} & \text{if } u < 1. \\ 5e \ln(e + u - 1) & \text{if } u \geqslant 1. \end{cases}$$

Find all values of k so that $N(u)$ is continuous at $u = 1$. Show your work carefully, and leave your answer(s) in exact form.

Problem 7 (Fall 2018 Exam 1)**.** The graph of a functions $Q(x)$ with domain $[-5, 5]$ is shown below.



(a) Find the numerical value of the following mathematical expressions. If the answer cannot be determined with the information given, write NI. If any of the quantities does not exist, write DNE.

(i) Find $\lim\limits_{x \to -1} Q(x)$.

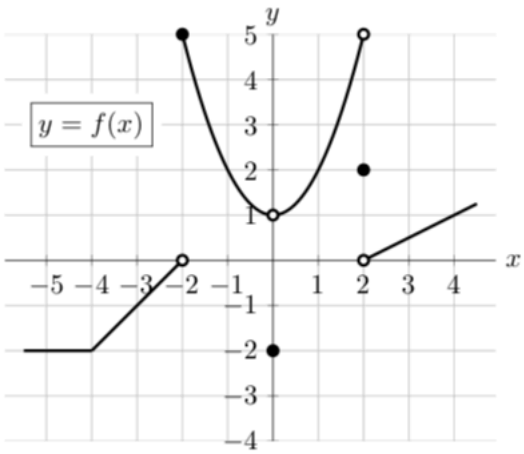
(ii) Find $\lim\limits_{w \to 2} Q(Q(w))$.

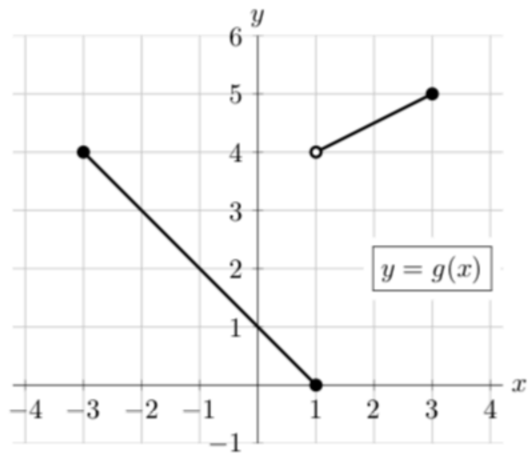
(iii) Find $\lim\limits_{x \to \infty} Q\left(\frac{1}{x} + 3\right)$.

(iv) Find $\lim\limits_{x \to \frac{1}{3}} xQ(3x - 5)$.

(b) For which values of $-5 < p < 5$ is $\lim\limits_{x \to p^-} Q(x) \neq Q(p)$?

Problem 8 (Winter 2017 Exam 1)**.** The graphs of the functions $f(x)$ and $g(x)$ are shown below.





Note that the graph of $f(x)$ is linear for $x < -2$ and $x > 2=$, and $g(x)$ is linear on $-3 < x < 1$ and $1 < x < 3$. For each of the following parts, find the given limit. If any of the quantities do not exist (including the case of limits that diverge to ∞ or $-\infty$), write DNE. If the limit cannot be information given, write NOT ENOUGH INFO. You do not need to show any work.

(a) Find $\lim\limits_{x \to -1} f(x)$.

(b) Find $\lim\limits_{t \to 2^-} 2(f(t) - 3)$.

(c) Find $\lim\limits_{x \to 1} f(x)g(x)$.

(d) Find $\lim\limits_{x \to \infty} f(e^{-x})$.

(e) Find $\lim\limits_{x \to 2^+} g^{-1}(x)$.