Worksheet 8

Warm-up questions

The average velocity of $v(t)$ in the interval $a \leqslant t \leqslant b$ is ————————.

The instantaneous velocity is _____.

Throwing a ball

Problem 1. A ball is thrown straight upward into the air at $t = 0$ seconds.

(a) Using the data given in the table below, sketch the height $y(t)$ measured in feet above the ground.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
$y(t)$	0	1.75	3	3.75	4	3.75	3	1.75	0

(b) Compute the average velocity of the ball over the interval $0 \leqslant t \leqslant 4$. Interpret your answer.

(c) Compute the average velocity $v(t)$ of the ball over the intervals indicated in the table below.

t	$[0, 0.5]$	$[0.5, 1]$	$[1, 1.5]$	$[1.5, 2]$	$[2, 2.5]$	$[2.5, 3]$	$[3, 3.5]$	$[3.5, 4]$
$v(t)$								

(d) Interpret the average velocities $v(t)$ of part (c) as difference quotients[1] and visualize all eight values of $v(t)$ in (c) with the help of triangles in the graph you drew.

Towards instantaneous velocity

Problem 2. Assume the following additional data on the ball's trajectory is available.

t	0.9	0.99	0.999	1	1.001	1.01	1.1
$y(t)$	2.79	2.9799	2.997999	3	3.001999	3.0199	3.19

(a) Compute the average velocity $v(t)$ of the ball over the intervals indicated in the table below.

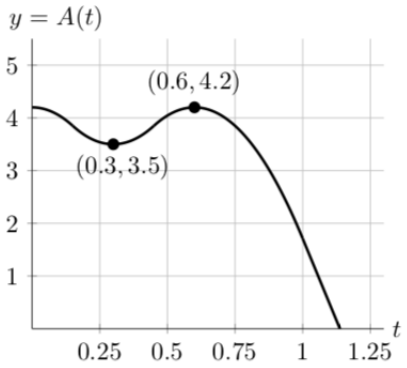
t	$[0.9, 0.99]$	$[0.99, 0.999]$	$[0.999, 1]$	$[1, 1.001]$	$[1.001, 1.01]$	$[1.01, 1.1]$
$v(t)$						

Use your answer to estimate the instantaneous velocity of the ball at time $t = 1$.

(b) Sketch the height $y(t)$ for $0.9 \leqslant t \leqslant 1.1$ using the additional data.

[1] Compare to the figure in page 5 of the text book.

Problem 3 (Winter 2018 Exam 1)**.** Tom organizes another meeting of his Science Club, but this time only Anne and John can make it. The meeting is at 2 pm, so they both start walking from their houses to Tom's at 1 pm. At 1:18 pm, Anne realizes she forgot her wallet, so she goes back home to get it before heading over to Tom's house. Annes distance in kilometers, $A(t)$, and John's distance in kilometers, $J(t)$, to Tom's house t hours after 1 pm are given by the graph and the table below. Assume that both of them walk along a straight line.



t	0	0.2	0.4	0.5	0.8	0.9
J(t)	5.5	4.3	3.2	2.8	0.8	0

(a) How many kilometers from Toms house is Anne's house?

(b) Estimate John's instantaneous velocity at 1:24 pm. Show all your computations. Include units.

(c) Rank John's average velocity over the time intervals

(I) $0.2 \leqslant t \leqslant 0.4$ (II) $0.5 \leqslant t \leqslant 0.9$ (III) $0.8 \leqslant t \leqslant 0.9$.

(d) What was the total distance travelled by Anne?

(e) On which of the following intervals is $A(t)$ invertible?

(I) $[0, 0.6]$ (II) $[0.3, 0.6]$ (III) $[0.1, 0.5]$ (IV) $[0.6, 1]$ (IV) $[0, 1]$.

Problem 4 (Fall 2017 Exam 1)**.** Han is playing with a balloon. He blows it up and then lets it go without tying it and watches it fly straight upwards away from him. Let B(t) be the distance, in inches, of the balloon from Han t seconds after he releases it. You may assume B is invertible on the interval shown below.

t (seconds)	0	0.2	0.6	0.8	0.9	1.2	1.4	1.6
B(t) (inches)	0	0.6	1.0	1.4	1.8	2.4	2.8	3.1

(a) What is the average velocity of the balloon over the first 0.8 seconds of its flight? Show your work and include units.

(b) Estimate the instantaneous velocity of the balloon 1.45 seconds after Han releases it. Show your work and include units.

(c) What is the average rate of change of B^{-1} over the interval [0.6, 1.4]? Show your work and include units.

(d) Over which of the following intervals could $B(m)$ be linear? Circle all possible intervals.

(I) $[0, 0.6]$ (II) $[0.6, 0.9]$ (III) $[0.9, 1.4]$ (IV) $[1.4, 1.6]$ (IV) None of these

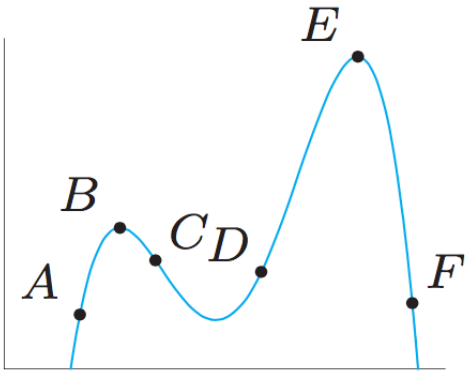
Problem 5. The position of an object is given by

$$s(t) = \begin{cases} -t^2 + 4t & 0 \le t < 3 \\ 3e^{-\frac{2}{3}(t-3)} & t > 3. \end{cases}$$

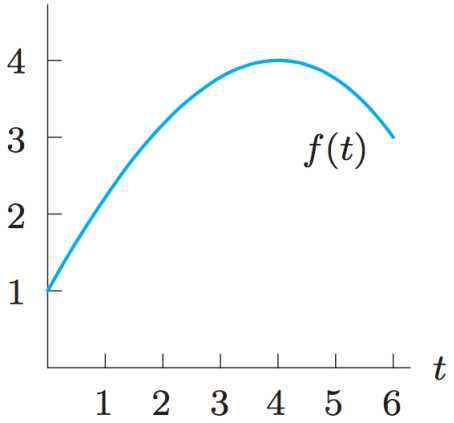
(a) What is the average velocity in the interval $[1, 4]$?

(b) What is the instantaneous velocity at $t = 4$?

Problem 6. For the function shown below, at what labeled points is the slope of the graph positive? Negative? At which labeled point does the graph have the greatest (i.e., most positive) slope? The least slope (i.e., negative and with the largest magnitude)?



Problem 7. The graph of $f(t)$ below gives the position of a particle at time t. List the following quantities in order, smallest to largest.



(a) A, average velocity between $t = 1$ and $t = 3$,

(b) B, average velocity between $t = 5$ and $t = 6$,

(c) C, instantaneous velocity at $t = 1$,

(d) D, instantaneous velocity at $t = 3$,

(e) E, instantaneous velocity at $t = 5$,

(f) F, instantaneous velocity at $t = 6$.