Worksheet 9

Warm-up questions

What is the definition of the derivative of the function $f(x)$ at $x = c$?

Problem 1 (Winter 2018 Exam 1)**.** Let $m(x) = (1+x^2)^{3x-4}$. Which of the limits below represents $m'(2)$? There is only one correct answer.

(a) $\displaystyle\lim_{h\to 0} \frac{(1+x^2)^{3x-4}+h-25}{h}$

(b) $\displaystyle\lim_{h\to 0} \frac{(1+h^2)^{3h-4}-25}{h}$

(c) $\displaystyle\lim_{h\to 0} \frac{(1+(2+h)^2)^{3h-4}-25}{h}$

(d) $\displaystyle\lim_{h\to 0} \frac{(1+(2+h)^2)^{3h+2}-25}{h}$

(e) $\displaystyle\lim_{h\to 0} \frac{(5+h^2)^{3h+2}-25}{h}$

(f) $\displaystyle\lim_{h\to 0} \frac{(1+h^2)^{3h+2}-25}{h}$

Problem 2 (Winter 2016 Exam 1)**.** Consider the function g defined by

$$g(x) = \begin{cases} \frac{1}{e^x-1} & \text{if } x < \frac{1}{2} \\ \cos(x^x) & \text{if } \frac{1}{2} \leqslant x < 5 \\ \frac{x^2}{(x-1)(6-x)} & \text{if } x \geqslant 5 \end{cases}$$

(a) Use the limit definition of the derivative to write an explicit expression for $g'(3)$. Your answer should not involve the letter g. Do not attempt to evaluate or simplify the limit.

(b) Find all vertical asymptotes of g, if there are any.

Problem 3 (Fall 2017 Exam 1)**.** Let

$$B(k) = e^{-4k^2}\tan(k+3).$$

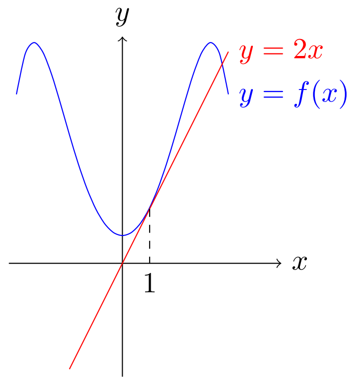
Use the limit definition of the derivative to write an explicit expression for $B'(5)$. Your answer should not involve the letter B. Do not attempt to evaluate or simplify the limit.

Problem 4 (Winter 2015 Exam 1)**.** Sebastian has chartered a helicopter which is rising straight up in the air, but he is scared of heights. Let $A(w)$ be Sebastians fear (in scared units) when he is w km above the ground. For $0 \leqslant w \leqslant 2$, a formula for $A(w)$ is given by

$$A(w) = \frac{w^2+2}{w^w+1}.$$

Use the limit definition of the derivative to write an explicit expression for the instantaneous rate of change of Sebastian's fear, in scared units per km, when he is 1.5 km above the ground. Your answer should not involve the letter A. Do not attempt to evaluate or simplify the limit.

Problem 5. The graph of the even function $f(x)$ is drawn below. We know that the tangent line to $f(x)$ at $x = 1$ is the line given by $y = 2x$.



Find the value of $f'(-1)$.

Problem 6 (Winter 2018 Exam 1)**.** Sketch the graph of a single function $y = f(x)$ satisfying all of the following conditions:

- The domain of $f(x)$ is the interval $-8 < x \leqslant 6$.
- $f(x)$ is continuous on the interval $-8 < x < -2$.
- $f'(-7) = 0$.
- $f(x)$ is decreasing and concave up for all x in the interval $-6 < x < -4$.
- The average rate of change of $f(x)$ is equal to 0.5 between $x = -5$ and $x = -2$.
- $f(0) = 2$ and $f'(0) = -1$.
- $\lim_{x \to 2^-} f(x) = f(2)$ and $\lim_{x \to 2^+} f(x) < \lim_{x \to 2^-} f(x)$.
- $f(x)$ has constant rate of change on the interval $3 \leqslant x \leqslant 6$.

Make sure that your graph is large and unambiguous.