Name:

Problem 1 (15 points)**.** The function $P(u)$ is given by the equation

$$P(u) = \begin{cases} e^u & u < 0 \\ u^2 - 3u + 1 & 0 \leqslant u < 2 \\ u - 3 & 2 \leqslant u < 4 \\ u^2 - 7u & u \geqslant 4 \end{cases}$$

For which values of u is $P(u)$ differentiable? Show all your work to justify your answer.

Exponential functions and polynomials are differentiable functions, $P(u)$ is differentiable for all u except possibly $u = 0, 2, 4$.

$\underline{u=0}$: P is continuous at 0, since $P(0) = \lim_{u\to 0^-} P(u) = \lim_{u\to 0^+} P(u)$:

$$\lim_{u\to 0^-} P(u) = \lim_{u\to 0^-} e^u = e^u\Big|_{u=0} = e^0 = \underline{1}$$

$$P(0) = \lim_{u\to 0^+} P(u) = \lim_{u\to 0^+} (u^2 - 3u + 1) = 0^2 - 3\times 0 + 1 = \underline{1}$$

But P is $\underline{\text{not}}$ differentiable at 0, since $\lim_{u\to 0^-} P'(u) = \lim_{u\to 0^-} e^u = 1 \neq -3 = \lim_{u\to 0^+}(2u-3) = \lim_{u\to 0^+} P'(u)$

$\underline{u=2}$ P is continuous at 2, since $P(2) = \lim_{u\to 2^-} P(u) = \lim_{u\to 2^+} P(u)$

$$\underline{P(2)} = \lim_{u\to 2^+} P(u) = u - 3\Big|_{u=2} = -\underline{1} = 4 - 6 + 1 = u^2 - 3u + 1\Big|_{u=2} = \lim_{u\to 2^-} P(u)$$

And P is differentiable at 2, since $\lim_{u\to 2^+} P'(u) = \lim_{u\to 2^+} 1 = 1 = 2u - 3\Big|_{u=2} = \lim_{u\to 2^-} P'(u)$

$\underline{u=4}$ P is NOT continuous at 4, since $P(4) = u^2 - 7u\Big|_{u=4} = 16 - 28 = -12$, but

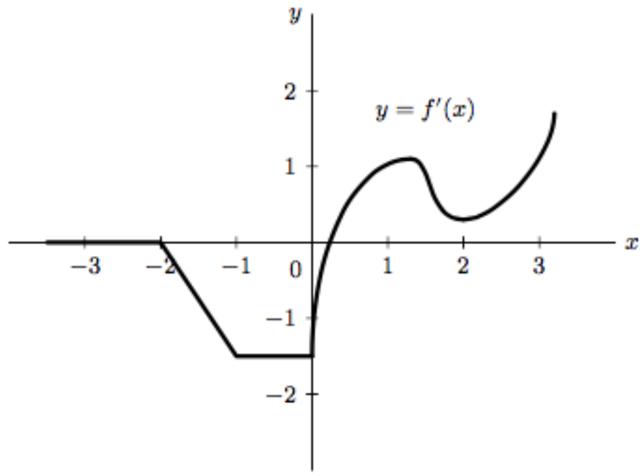
$$\lim_{u\to 4^-} P(u) = \lim_{u\to 4^-} (u-3) = 1 \neq -12$$

then P $\underline{\text{cannot}}$ be differentiable at 4, since differentiable $\Rightarrow$ Continuous.

$\underline{\text{Conclusion}}$ P is differentiable for all values of u except $u = 0$ and $u = 4$

$\underline{\text{other way to answer}}$: P is differentiable at $(-\infty, 0) \cup (0, 4) \cup (4, \infty)$

Problem 2 (15 points)**.** Below is the graph of $f'(x)$, the **derivative** of the function $f(x)$. Note that $f'(x)$ is zero for $x \leqslant -2$, linear for $-2 < x < -1$, and constant for $-1 < x < 0$.



For each of the following, circle all of the listed intervals for which the given statement is true over the entire interval. If there are no such intervals, circle NONE. You do not need to explain your reasoning.

(a) $f'(x)$ is increasing.

$-2 < x < -1$ $\quad 0 < x < 1$ $\quad 1 < x < 2$ $\quad 2 < x < 3$ $\quad$ NONE

(b) $f'(x)$ is concave up.

$0 < x < 1$ $\quad 1 < x < 2$ $\quad 2 < x < 3$ $\quad$ NONE

(c) $f(x)$ is increasing.

$-2 < x < -1$ $\quad -1 < x < 0$ $\quad 0 < x < 1$ $\quad 1 < x < 2$ $\quad 2 < x < 3$ $\quad$ NONE

(d) $f(x)$ is linear but not constant.

$-3 < x < -2$ $\quad -2 < x < -1$ $\quad -1 < x < 0$ $\quad 0 < x < 1$ $\quad 1 < x < 2$ $\quad 2 < x < 3$ $\quad$ NONE

(e) $f(x)$ is constant.

$-3 < x < -2$ $\quad -2 < x < -1$ $\quad -1 < x < 0$ $\quad 0 < x < 1$ $\quad 1 < x < 2$ $\quad 2 < x < 3$ $\quad$ NONE

Problem 3 (Bonus question: 3 points)**.** When are Eloísa's office hours?