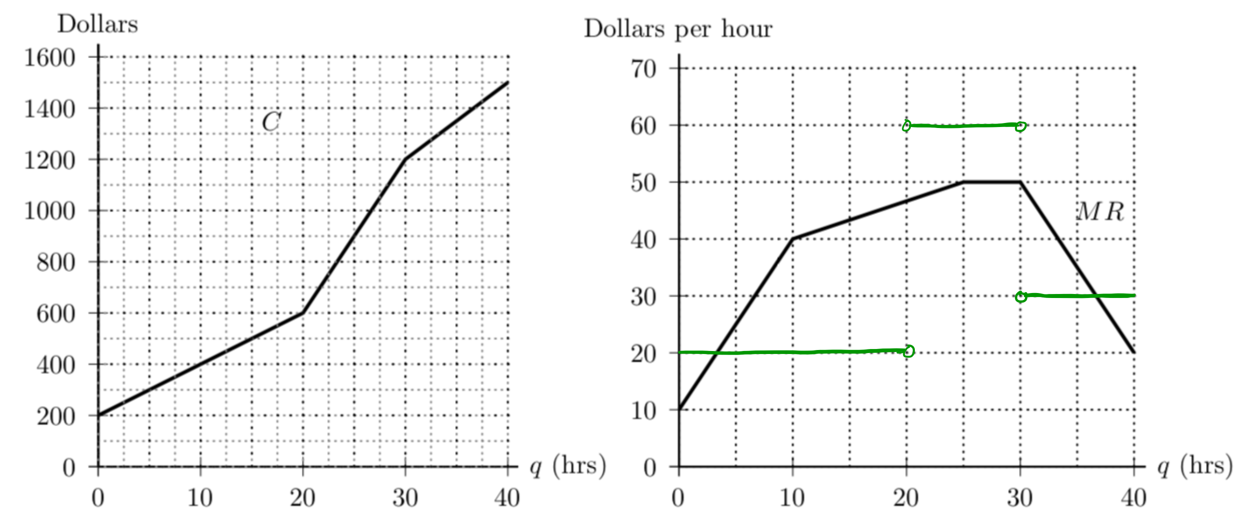
Name: Solutions

Problem 1 (10 points)**.** The now infamous Roderick has been dethroned. Before Erin returns to the University of Michigan, she visits Roderick to hear his side of the story. He encourages her to share his story. Erin is in fact quite a good storyteller, so she begins to consider a career as a traveling storyteller. She decides to charge clients for her time (in hours).

(a) (3 points) Shown below are graphs of the cost, C, and marginal revenue, MR, of Erin's potential storytelling business. Note that both graphs are continuous and piecewise linear. Carefully sketch the graph of Erin's marginal cost function on the same axes as the given graph of her marginal revenue. (That is, draw the graph of marginal cost on the set of axes on the right.)



(b) (1 point) What is Erin's fixed cost?

\$200.

(c) (2 points) For what values of $0 < q < 40$ is MR=MC?

$q = \frac{10}{3}, \frac{110}{3}$

For $0 < q < 10$, $MR(q) = 3q + 10$

For $30 < q < 40$, $MR(q) = -3q + 140$

- $3q + 10 = 20 \iff q = \frac{10}{3}$
- $-3q + 140 = 30 \iff q = \frac{110}{3}$

(d) (2 points) What are the critical points of Erin's profit $\pi(q)$ for $0 < q < 40$?

$q = \frac{10}{3}, 20, 30, \frac{110}{3}$

(e) (Bonus question: 3 points) Erin is very busy studying for her Math 115 class, so can only spend at most 40 hours on her story telling business. How many hours of work as a traveling storyteller should she do in order to maximize her profit?

$\pi'(q) = MR(q) - MC(q)$

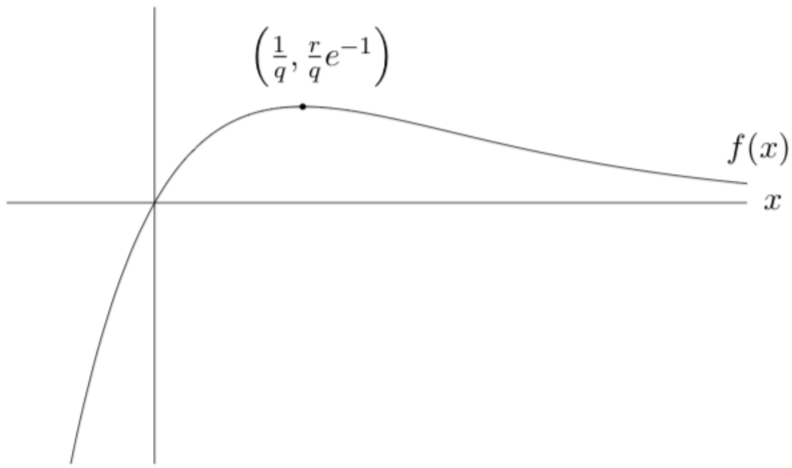
Need to consider: $q = 0, \frac{10}{3}, 20, 30, \frac{110}{3}, 40$.

π increases for $\frac{10}{3} < q < 20$ and $30 < q < \frac{110}{3}$, so the answer is not $\frac{10}{3}$ nor 30

π decreases for $0 < q < \frac{10}{3}$, $20 < q < 30$ and $\frac{110}{3} < q < 40$, so the answer is not 40

the answer is among $\boxed{0, 20, \frac{110}{30}}$ $\Rightarrow$ we will see later why the answer is 20.

Problem 2 (10 points)**.** Below is the graph of the function $f(x) = rxe^{-qx}$, where $r > 1$ and $q > 1$ are constants, which passes through the origin and has a local maximum at the point $P = (\frac{1}{q}, \frac{r}{q}e^{-1})$. Note that $f'(x) = r(1 - qx)e^{-qx}$ and $f''(x) = rq(qx - 2)e^{-qx}$.



(a) Use calculus to carefully justify that f has a local maximum at P.

$f'(\frac{1}{q}) = r(1 - q\frac{1}{q})e^{-q} = 0$ $\qquad$ $f''(\frac{1}{q}) = -\underbrace{rq}_{>0}\, e^{-q} < 0$

By the 2^{nd} derivative test, f has a local maximum at P

In the following questions, if $f(x)$ does not have a global maximum on this domain, say *no global maximum*, and similarly if $f(x)$ does not have a global minimum.

(b) What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain $[0, 1]$?

$f'(x) = 0 \Leftrightarrow r(1-qx)e^{-qx} = 0 \Leftrightarrow 1 = qx \Leftrightarrow x = \frac{1}{q} \Rightarrow \frac{1}{q}$ only critical point

By the Extreme Value Theorem, f does have both a global maximum and a global minimum on $[0,1]$

$f(0) = 0$, $f(\frac{1}{q}) = \frac{r}{q}e^{-1} > 0$, $f(1) = re^{-q} > 0$ $\qquad$ Note: f decreasing for $\frac{1}{q} < x$, since $f'(x) = + \cdot - \cdot + = -$

Global min at 0, Global max at $\frac{1}{q}$

(c) What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain $(-\infty, \infty)$?

f has only one critical point, at $\frac{1}{q}$

f' continuous on $(-\infty, \infty)$, $f'(x) = + \cdot + \cdot + = +$ for $x < \frac{1}{q}$, so f increasing on $(-\infty, \frac{1}{q})$

Have seen: f decreasing on $(\frac{1}{q}, \infty)$

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \underset{\downarrow\, -\infty}{rx}\ \underset{\downarrow\, +\infty}{e^{-qx}} = -\infty$

$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{rx \to \infty}{e^{qx} \to \infty} = 0$

dominates

Global max: At $x = \frac{1}{q}$

Global min: None