Name: Solutions

Problem 1 (4 points)**.** Circle all the true statements, no justification necessary.

(a) D_7 contains an element of order 3.

(b) $\mathbb{Z}_4^\times \cong \mathbb{Z}_3^\times$. $\cong \mathbb{Z}_2$

(c) If G and H are groups of order 17, $G \cong H$. $\cong \mathbb{Z}_{17}$

17 is prime!

(d) Faithful actions have no fixed points.

D_4 acting on the $\square$ has a fixed point, and this is a faithful action.

Problem 2 (3 points)**.** State Lagrange's Theorem.

Let G be a group and K a subgroup of G.

then $$|G| = \underbrace{[G:K]}_{\text{index of } K \text{ in } G} |K|$$

Problem 3 (3 points)**.** True or false: given any group G of order 16 and any group H of order 24, there is no injective homomorphism $G \longrightarrow H$.

True. Suppose $f: G \to H$ is an injective group homomorphism.

then $G \cong \operatorname{im} f$ is a subgroup of H of order 16.

By Lagrange's theorem, $16 \mid |G| = 24$, which is false.

therefore, no such f exists.