Name: Solutions

Problem 1 (6 points)**.** Circle all the subsets that are ideals of the given ring.

a) The subset of $\mathbb{Z}$ of all even numbers.

b) The subset of $\mathbb{Z}$ of all odd numbers. $1+1=2$

c) Matrices in $M_2(\mathbb{Z})$ with even entries.

d) All linear combinations of 6 and 9 in $\mathbb{Z}$. Aka (3)

e) $\{0\} \subseteq \mathbb{R}[x]$. $\{0_R\}$ is always an ideal in R

f) The subset of $\mathbb{Z}[x]$ all $p \in \mathbb{Z}[x]$ with $p(0) = 1$. $1+1=2 \notin$

Problem 2 (4 points)**.** Consider the polynomials $p(x) = x^2 + 7x + 6$ and $q(x) = x^2 - 5x - 6$ in $\mathbb{Q}[x]$. Use the Euclidean algorithm to find their greatest common divisor.

$$x^2 + 7x + 6 = 1 \cdot (x^2 - 5x - 6) + \underbrace{12x + 12}_{= 12(x+1)}$$

$$x^2 - 5x - 6 = (x-6)(x+1) + 0$$

$$\Longrightarrow \gcd(p,q) = x+1$$

And the linear combination we are searching for is:

$$x + 1 = \frac{1}{12}\left(x^2 + 7x + 6\right) - \frac{1}{12}\left(x^2 - 5x - 6\right)$$