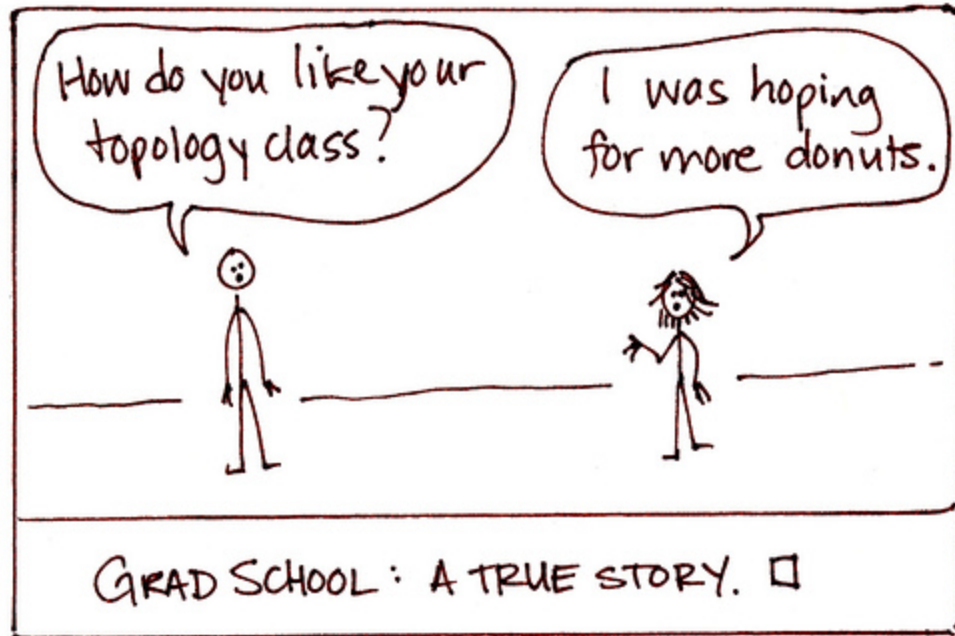


Math 871 Midterm 2
Fall 2026

Name:



2007

@COURTNEY GIBBONS

(Fun fact: Courtney Gibbons got her PhD from UNL!)

Instructions: All of your answers require a complete proof unless otherwise stated. You may use any results proved in the course notes, the problem sets, or the worksheets, except for the specific question being asked. You should clearly state any facts you use. You may use anything stated in one problem to solve a different problem. Show all your work, and write clearly and using complete sentences. If you run out of space, continue your work in the last few pages, which were left blank for this purpose, and clearly indicate you have done so.

Best of luck!

Problem 0. The questions on this page require **no proof or explanation**.

a) Define continuous function.

b) Define homeomorphism.

c) Give an explicit basis for the product topology on $\mathbb{R}^\omega$.

d) Give an example of a topological space that is Hausdorff.

Problem 1.

a) Show that every metrizable space is Hausdorff.

b) Show that $\mathbb{R}$ with the finite complement topology is not a metrizable space.

Problem 2. Let X and Y be topological spaces and let $f: X \longrightarrow Y$ be any function.

- a) Show that if f is continuous, then f sends convergent sequences to convergent sequences, meaning that for every sequence $(x_n)_{n \geq 1}$ of points in X converging to x , the sequence $(f(x_n))_{n \geq 1}$ converges to $f(x)$.

- b) We say X is **Fréchet-Urysohn** if for every subset $S \subseteq X$ and every $s \in \overline{S}$, there exists a sequence (s_n) of points in S converging to s . Show that if X is Fréchet-Urysohn and f sends convergent sequences to convergent sequences, then f is continuous.

Problem 3. Choose **one** of the following two problems:

- A. Endow $\mathbb{R}$ with the standard topology. Show that the product and uniform topologies on $\mathbb{R}^\omega$ do not coincide.
- B. Show that the arbitrary product of Hausdorff spaces is Hausdorff in the product topology.

Problem 4. Choose **one** of the following two problems.

- A. Let $f: X \rightarrow Y$ and $g: X \rightarrow Y$ be continuous functions with Y a Hausdorff space. Show that $C = \{x \in X \mid f(x) = g(x)\}$ is a closed subset of X .
- B. Let X and Y be topological spaces. A function $f: X \rightarrow Y$ is **locally constant** if for all $x \in X$ there exists an open neighborhood U of x such that $f|_U$ is constant. Show that every locally constant function is continuous.

You can use this page to continue work from previous pages.

You can use this page to continue work from previous pages.

You can use this page to continue work from previous pages.