Math 871 Topology I
Problem Set 10

1. Let A and B nonempty open subsets of a topological space Z. Prove that if both $A \cap B$ and $A \cup B$ are connected, then each of A and B is connected.

2. Let X be a connected space and endow $\mathbb{R}$ with the standard topology and $[0,1] \subseteq \mathbb{R}$ with the subspace topology. Let $f, g \colon X \longrightarrow [0,1]$ be continuous functions and assume that f is surjective. Show that there exists $x \in X$ with $f(x) = g(x)$.

3. Endow $\mathbb{R}$ with the standard topology. Show that for all continuous maps $f \colon S^1 \longrightarrow \mathbb{R}$, there exists $x \in S^1$ such that $f(x) = f(-x)$.

4. Let X and Y be topological spaces and $f \colon X \longrightarrow Y$ a continuous function. Define an equivalence relation on X by $x_1 \sim x_2$ if and only if $f(x_1) = f(x_2)$. Show that if X is connected and Y is Hausdorff, then the quotient space $X/\sim$ with the quotient topology is both connected and Hausdorff.