

Math 871 Topology I

Problem Set 4

Instructions: You will submit your work on gradescope as a pdf. If you choose to handwrite your work, please write neatly, and make sure everything is clearly legible in the scan. As a matter of good proof writing style, use complete sentences and correct grammar. You may use any result proven in class or in a homework problem, provided you reference it appropriately by either stating the result or its name. Please do not refer to theorems by their number in the course notes, as that can change.

You are strongly encouraged to work with your classmates and to discuss the problems with me. However, you should write your own final solutions. Never submit something for grading that you do not completely understand. See the course syllabus for the AI policy for this class.

1. Find the closure of $(0, 1)$ in $\mathbb{R}_\ell$.
2. Let X be a topological space and consider $A \subseteq B \subseteq X$ with the subspace topologies.
 - (a) Show that if A is dense in B and B is dense in X , then A is dense in X .
 - (b) Show that if A is not dense in B , then A is not dense in X .
3. Let X and Y be topological spaces and consider $A \subseteq X$ and $B \subseteq Y$. Show that in $X \times Y$ with the product topology,

$$\overline{A \times B} = \overline{A} \times \overline{B}.$$

4. Let X be a topological space and $A \subseteq X$. The **boundary** of A is defined by

$$\text{Bd}(A) = \overline{A} \cap \overline{X \setminus A}.$$

- (a) Show that A^{int} and $\text{Bd}(A)$ are disjoint.
- (b) Show that $\overline{A} = A^{\text{int}} \cup \text{Bd}(A)$.
- (c) Show that $\text{Bd}(A) = \emptyset$ if and only if A is a clopen set.