Math 871 Topology I
Worksheet 1

1. Let $(X, \mathcal{T})$ be a topological space, and let A be a subset of X. Suppose that for every $x \in A$, there exists an open set U such that $x \in U \subseteq A$. Show that A is an open set.

2. Prove that the finite complement topology on any set X is indeed a topology.

3. Show that the **infinite ray topology** on $\mathbb{R}$, with open sets

$$\mathcal{T} = \{(-\infty, a) \mid a \in \mathbb{R}\} \cup \{\emptyset, \mathbb{R}\}.$$

   is indeed a topology.

   Hint: you may need to use the Completeness Axiom for $\mathbb{R}$, which says that

   Every nonempty, bounded above subset of $\mathbb{R}$ has a least upper bound.

4. Let X be a set and let $\mathcal{T}$ and $\mathcal{T}'$ be two topologies on X. We say that $\mathcal{T}$ is **finer** than $\mathcal{T}'$, or equivalently that $\mathcal{T}'$ is **coarser** than $\mathcal{T}$, if $\mathcal{T}' \subseteq \mathcal{T}$. Note that two topologies can also be **incomparable**. Compare the following topologies on $\mathbb{R}$:

   (a) $\mathcal{T}_1$ the standard topology.
   (b) $\mathcal{T}_2$ the finite complement topology.
   (c) $\mathcal{T}_3 = \{(-\infty, a) \mid a \in \mathbb{R}\} \cup \{\emptyset, \mathbb{R}\}$ the infinite ray topology.
   (d) $\mathcal{T}_4 = \{(a, \infty) \mid a \in \mathbb{R}\} \cup \{\emptyset, \mathbb{R}\}$.
   (e) $\mathcal{T}_5$ the discrete topology.
   (f) $\mathcal{T}_6$ the indiscrete topology.

5. Show that the line with two origins is a topological space.