

## Worksheet 4: Gorenstein rings

## OUR ZOO OF RINGS

- (1) Let  $k$  be a field.
- (a) Give an example (with proof) of a complete intersection ring  $A$  that is not regular.
- (b) Show that
- $$B = k[[x, y, z]]/(x^2 - y^2, x^2 - z^2, xy, xz, yz)$$
- is Gorenstein but not a complete intersection.
- (c) Let  $k$  be any field. Show that
- $$C = k[[x, y]]/(x^2, xy, y^2)$$
- is Cohen-Macaulay but not Gorenstein.
- (d) Give an example (with proof) of a noetherian local ring  $B$  that is not Cohen-Macaulay.

## GORENSTEIN RINGS

- (2) Is  $\mathbb{Z}[x, y]/(x^2 - y^2)$  a Gorenstein ring?
- (3) Let  $k$  be a field. For each of the following rings, determine which ones are Gorenstein.
- (a)  $A = k[[x]]/(x^3)$ .
- (b)  $B = k[[x, y]]/(x^2, y^3)$ .
- (c)  $C = k[[x, y]]/(x^3, x^2y, xy^2, y^3)$ .
- (d)  $D = \mathbb{Z}/(4)$ .
- (4) Let  $k$  be a field. For each of the following rings, determine which ones are regular, complete intersections, Gorenstein, or Cohen-Macaulay.
- (a)  $k[[x, y, z]]$
- (b)  $k[[x, y, z]]/(x + y + z)$
- (c)  $k[[x]]/(x^2)$
- (d)  $k[[x, y]]/(xy)$
- (e)  $k[[x, y]]/(x^2, xy)$
- (f)  $k[[x, y, z]]/(x^3, y^3, z^3)$
- (g)  $k[[x, y, z]]/(x^3 + y^3 + z^3)$
- (h)  $k[[x, y, z]]/(xy^3, y^4, y^3z)$
- (i)  $k[[x, y, z, a, b]]/(x, y, z) \cap (a, b)$
- (j)  $k[[x, y, z, a, b]]/(x^2, y^3, z^4) \cap (a^5, b^6)$
- (k)  $k[[x, y, z, a, b]]/(x, y, z) \cap (a, b)$