Problem Set 4

To solve these problems, you are not allowed to use any additional Macaulay2 packages besides the `Complexes` package and the ones that are automatically loaded with Macaulay2.

Problem 1. Let k be any field and consider the prime ideal in $k[x, y, z]$

$$P = (x^3 - yz, y^2 - xz, z^2 - x^2y)$$

defining the curve parametrized by (t^3, t^4, t^5). Give (with proof!) two different ideals J such that $P^{(n)} = (P^n : J^\infty)$ for all $n \geqslant 1$, and test your proposed ideals J in Macaulay2 with your own choice of k and n.

Problem 2. Let R be a finitely generated k-algebra, and P a prime ideal in R. Show that $P^{\langle n \rangle}$ is P-primary for all $n \geqslant 1$.

Problem 3. Let k be any field, $d \geqslant 1$, and $R = k[x_1, \dots, x_d]$. Let

$$I = \left(\prod_{i \neq j} x_i \mid 1 \leqslant j \leqslant d \right)$$

be the monomial ideal generated by all the squarefree monomials of degree $d - 1$.

a) Find an irredundant primary decomposition of I. What is the height of I?

b) Find a set of generators for $I^{(n)}$ for each $n \geqslant 1$.

c) Show that $I^{(2n-2)} \nsubseteq I^n$ for all $n < d$.

Problem 4. Let I be a squarefree monomial ideal in $R = k[x_1, \dots, x_d]$ and $\mathfrak{m} = (x_1, \dots, x_d)$. Show that $I^{(n+1)} \subseteq \mathfrak{m} I^{(n)}$ for all $n \geqslant 1$.

Problem 5. The **complete graph** K_n is the simple graph on n vertices that has all the possible edges. Let $I = I(K_n)$ be the edge ideal of K_n with $n \geqslant 3$. Find generators for $I^{(s)}$ for all s and show that $I^{(s)} \neq I^s$ for infinitely many values of s.