The homotopy Lie algebra of a Tor-independent tensor product

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SET UP 
$$(R, m, K)$$
 LOCAL (OMM,  $I_1, I_1 \subseteq M$  IDEALS  
 $S_1 = \frac{R}{I_1}$ ,  $S_2 = \frac{R}{I_2}$   $S = S_1 \approx S_2 = \frac{R}{I_1 + I_2}$ 

QUESTION: HOW DO THE HOMOLOGICAL PROPERTIES OF R, SI, S2, S RELATE )

IN A '75 PADER AURAMOU STUDIED THE RELATION BETWEEN THE TOR ALGEBRAS OF THESE RINKS

THM(TATE, GUILIKSEN, SCHOELLER)

THERE IS A MINIMAL DGA RESOLUTION OF K IR

DEF: Tork(K,K):= FRK

THM( AURAMON '75):

IF R HOULAR,  $I_1, I_2 \leq m^2$  AND  $I_1I_2 = I_1 \cap I_2$ , THEN  $Tor^{S}(K, K) \leq Tor^{S}(K, K) \otimes Tor^{S_2}(K, K)$  $Tor^{R}(K, K)$  DEF:  $Q: R \rightarrow S$  IT INDUCES A MAP  $F_R \rightarrow F_S$   $T_{\sigma}^{q}(k,k): \Gamma_{\sigma}^{k}(k,k) \rightarrow T_{\sigma}^{s}(k,k)$  q is <u>SMACC</u> IF  $\Gamma_{\sigma}^{q}(k,k)$  is inservice THMC AURAMON (78):  $R \stackrel{q}{=} JS_{r}$  $q_{2} \int J$ 

$$IF ONE OF THE P: IS SMALL AND  $Tor_{so}^{R}(S, S) = 0 - THEN$   
$$Tor^{S}(K, K) = Tor^{S}(K, K) \otimes \tilde{lor}^{S}(K, K)$$
  
$$Tor^{R}(K, K)$$$$

OUR WORK: WE INDESTIGATED HOW THE HOMOTOPY LIE ALGEBRAS OF- THRSE RWGS RELATE R, S, S, S2, S

CONSTRUCTION: F-SK F MINIMAL DEA RES OF K Derg (F,F) = { = { (Young (F,F) | = SATISFIES THE S LEIBNIT AULT

DEF: THE MOMOTORY LIE ALG JT(K): H ( Der, (F, F))

EX: q: R - >> S R KEG => q Acmost Small

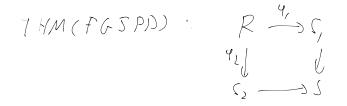
THM(FGJPP) 
$$R \xrightarrow{Y_1} G_1$$
  
 $Y_2 \bigcup U$   
 $S_2 \xrightarrow{\rightarrow} S$ 

 $\begin{aligned} \text{IF ONE OF THE } f_{C} & \text{IS ALMOST SMALL AND } & \text{Torse}^{R}(S_{1}, S_{2}) \neq 0, \\ & \text{THEA} \end{aligned}$   $\begin{aligned} \text{TCS} & \equiv & \text{TC}(S_{1}) + \text{TCS}_{1} \\ & \text{TCR} \end{aligned}$ 

1) A SEMIFREE EXTENSION OF R IS A DOA REX] OBTAINED B-1 ADDING EXTERIOR VARIABLES IN ODD DEG AND POLYNOMIAL VARIABLES IN EVEN DEG

GULLIKSEN MINIMALITY LET Q C P SS R P SS S QEWJ MIN MOD FOR P REXJ MIN MOD FOR P

DEF: f 15 e-GULLIKSEN MINIMAL IF QEX, WI FATISFIES  $\begin{pmatrix} - & -\frac{\partial e}{\partial x} \\ - & -\frac{\partial e}{\partial x} \\ - & -\frac{\partial e}{\partial x} \end{pmatrix}$ 



 $IF = P, AND P_2 ARE GULLINSEN MWIMAN AND POR_{(S_1, S_2)=0}$ THEN  $TT(S) \cong TT(S_1) \times T(S_2)$ T(R)

THM(FGJDD) IF (HAR K= O AND PORSO(SI,S2)=U

THEN  $\pi(S) \cong \pi(S, ) \times \pi(S_{L})$  $\pi(K)$ 

RECALL: 8 LIE ALG US =  $\frac{\overline{18}}{(XBY - C-1)^{WIIYI}Y \otimes X - E \times iA}$ , ×, Y eg)

FACT:  $Hom_{k}(U\pi(R), \kappa) \cong Tor^{R}(\kappa, \kappa)$ FACT: · V PRESERVES PULLBACKS ·  $Hor_{K}(-, \kappa)$  SENDS PULLBACKS TO PUSHOUTS  $\begin{array}{c} (ORG(CARY (FGJPD)) \\ R \xrightarrow{4} JS, \\ Y_{2}J \\ I \\ S_{2} \xrightarrow{1} S \end{array}$   $\begin{array}{c} WITIX & Tor_{S_{0}}^{R}(S_{1}, S_{2}) \xrightarrow{1} G \\ S_{1} \xrightarrow{1} S \end{array}$ 

1) 9: 15 ALMOST SMALL PCK SOME i 2) 9,92 ARE GULLINSER MINIMAL 3) (HAR K=0

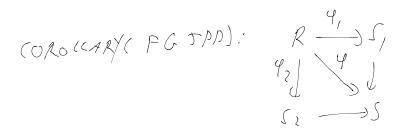
$$THFN \quad Tor^{S}(H, H) \cong Tor^{S'}(H, K) \otimes \tilde{for}^{S_{2}}(H, K)$$
$$for^{R}(H, H)$$

GOLODNESS REXI: q: R-DDS REXI-DS MW MODEL THE FIBER OF Q IS F<sup>Q</sup> = REXI & K R

THEN  $\pi(F^{q}) \cong \pi(F^{q}) \times \pi(F^{q})$ THEN  $\pi(F^{q}) \cong \pi(F^{q}) \times \pi(F^{q})$  AVRAMOU: f is GOIOD (=)  $\pi(f^{q})$  is FAFR (OROICARY: IF  $for_{26}^{k}(S_{1}, G_{2}): 0 =) f(R \rightarrow S)$  is NOT GOLOD (TABLE COHOMOLOGX

DEFILIF A 15 A URADED CONNECTED K-ACG DEDTH A: INF & M201 EXEM(K,A) = 0 UM(R) " THM ( AURAMOV-URLICHE),' IF DEPTH EXER(K,K)=2 THEN EXER(K,K) HAS A SIMPLE STRUCTURE

THMC FERRARD: IF RIGOREN(TEN AND DEPTH EXER(KIH) J=2  $\Gamma HEN$  $\overline{E_{A}(K_{i}K_{i})} \cong E_{A}(K_{i}K_{i}) \propto 2^{1-o(imR)} E_{A}(K_{i}K_{i})^{*}$ 



IF  $q_1, q_2$  MINIMAL COHEN INRES  $S_1, S_2$  SINGULAR

$$T\sigma_{s_0}^{\mathcal{R}}(S_1, S_2) = 0$$

$$T H \in \mathcal{N} \qquad D \in PT H \quad \mathcal{E}_{\mathcal{R}}^{\mathcal{C}}(K, H) \cong 2$$

$$PRooF: \quad D \in PT H \quad \mathcal{E}_{\mathcal{R}}^{\mathcal{C}}(K, K) = D \in PT H \quad U \pi (F^{\mathcal{Q}})$$

$$= D \in PT H \quad U \pi (F^{\mathcal{Q}}) + D \in PT H \quad U \pi (F^{\mathcal{Q}})$$

$$= D \in PT H \quad \mathcal{E}_{\mathcal{R}}^{\mathcal{C}}(H_1 H) + D \in PT H \quad \mathcal{E}_{\mathcal{R}}^{\mathcal{C}}(H_1 H)$$

$$= 1 + 1 = 2$$

$$\blacksquare$$

PROP (FGJPP, MORE - JORGENSEN)  
Torso (S, S2)  

$$f$$
 is GOR (S)  $f_{1}, f_{2}$  GOR  
 $f$  is GOR (S)  $f_{1}, f_{2}$  GOR  
 $poinCARE$  SERIES  
 $rorologically (FGJPP)$ :  
 $R \xrightarrow{4}{} S_{1}$   $roron R (S_{1}, S_{2}) = 0$   
 $S_{1} \xrightarrow{5} S$ 

 $\frac{1F}{T} \frac{AT}{K} \frac{1FAST}{K} \frac{1}{K} \frac{1}{K$ 

PROOF: 
$$(1: R \rightarrow S) \land (mosr small)$$
  
 $(=) \pi^{2}(S) \rightarrow \pi^{2}(R) \qquad (S \qquad SUKT)$   
 $B(OVR \qquad MIM \qquad \pi^{2}(S) = \pi^{2}(S_{1}) \propto \pi^{2}(S_{2})$   
 $\pi^{2}(R) \rightarrow \pi^{2}(S) \rightarrow \pi^{2}(S_{1}) \times \pi^{2}(S_{2}) \rightarrow \pi^{2}(R) \rightarrow 0$   
 $Almosi
find (C) = Gin \pi^{1}(S_{1}) + Gim \pi^{1}(S_{2}) - Gin \pi^{1}(R)$   
 $(=) L$ 

$$(HE(K \quad \operatorname{For}_{S}^{R}(S_{1},S_{2}) \rightarrow 0 \quad GIUES \quad Y_{6}U \quad THE \quad GOURCHY \quad FOR$$

$$\widehat{c} = I$$

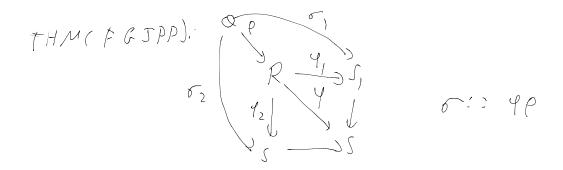
$$FA(T: \quad P_{K}^{R}(E) = \int C \quad (I + E^{2i-1}) \quad Gim \quad \pi^{2i}(R) \quad A$$

$$\widehat{c} = I$$

$$THM(QUILLEN): IF (MARK=0)$$

$$q: R \rightarrow S S , THEN$$

$$\pi(F^{q}) \cong \Pi(S|R'K)$$



 $\begin{aligned} &|F \ (HARK=0 \ (OR \ IF \ \Psi_{1}, \Psi_{2} \ ARF \ P-GULLIKSEN MINIMUL) \\ &AND \ \mathcal{T}_{S_{0}}^{\mathcal{R}}(S_{1}, S_{2})=0, \quad F \not \neq \mathcal{N} \\ &\mathcal{T}_{S_{0}}^{\mathcal{R}}(F^{\sigma}) \cong \mathcal{T}_{S_{0}}(F^{\sigma}) \xrightarrow{\mathcal{R}}(F^{\sigma}) \\ &\mathcal{T}_{S_{0}}(F^{\sigma}) \cong \mathcal{T}_{S_{0}}(F^{\sigma}) \xrightarrow{\mathcal{R}}(F^{\sigma}) \end{aligned}$ 

THM(QUILLEN): IF CHARKES AND A, B ARE LOCAL NOEYD K-ALG WITH RESIDUE FIELDE AND ESSENTIALLY OF FINITE TYDE, THEN D(KIASBJK) = D(KIASK) × D(KIBJK)