Homological Methods in Commutative Algebra

Problem Set 4

Throughout, all rings all commutative and noetherian, and all modules are finitely generated.

Problem 1. Let k be a field and consider the monomial ideal

$$I = (xy, xz, yz) \subseteq Q = k[\![x, y, z]\!].$$

Set $R = Q/I$. Calculate the algebra $\operatorname{Tor}^Q(R, k)$ by giving explicit basis elements for each $\operatorname{Tor}_i^Q(R, k)$ and describing the product structure.

Problem 2. Let k be a field, and

$$I = (x^2, xy, yz, zw, w^2) \subseteq Q = k[\![x, y, z, w]\!].$$

a) Use the Taylor resolution for I to find an explicit basis for $\operatorname{Tor}_3^Q(R, k)$.

b) Show that
$$\operatorname{Tor}_1^Q(R, k) \cdot \operatorname{Tor}_3^Q(R, k) = 0.$$